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Threshold Behaviour of Meson-Nucleon-$S_{11}(1535)$ Vertexfunctions and Determination of the $S_{11}(1535)$ Mixing Angle

F. Kleefeld*

University of Erlangen–Nürnberg, Institute for Theoretical Physics III, Staudtstr. 7, D-91058 Erlangen, Germany

Abstract

A new method for the determination of the spin-1/2-3/2-mixing angle and the range parameter of the quarkmodel wavefunction of the resonance $S_{11}^-(1535)$ is presented. The method ist based on a quantitative calculation of the total cross section of $pp \to pp\eta$ at threshold. The quantitative on-shell treatment of ISI and FSI is discussed.

INTRODUCTION

Nearly all calculations for the determination of mixing angles between different spin-components of baryon wavefunctions are based on spectroscopic models. Beside their advantages these models also contain some problems which are not dissolved yet.
The goal of the presented work is an independent way of the determination of mixing angles and range parameters in the harmonic oscillator description of nucleonic resonances within the nonrelativistic quark model. Although the approach in a first step is applied to the spin–1/2–3/2–mixing angle θ in the negative parity resonance $S_{11}(1535)$ (the underlying nature of this resonance is at the moment object to various speculations), the approach also can be applied to many other resonances.
The idea of the presented method is as follows: In a first step one has to calculate the total cross section of a process which is dominated by the resonance under consideration in such a way that the only remaining free parameters of the calculation are the mixing angles and the range parameters of the harmonic oscillator wavefunctions of the resonance and the involved ground state nucleons. In a second step the parameters are obtained by adjusting the calculated to the experimentally measured cross section.
One process dominated mainly by the $S_{11}(1535)$ is the reaction $pp \to pp\eta$ at threshold. In order to determine the desired mixing parameters one first has to get a *quantitative* understanding not only of the *short range dynamics* of the process, but also of the *initial and final state interactions* (ISI and FSI) between the in- and outgoing particles. Main investigations along this line have been performed in [1] and references therein.

THE REACTION $pp \to pp\eta$ AT THRESHOLD

The reaction $pp \to pp\eta$ at threshold is described within a relativistic meson-exchange model. The virtual $S_{11}(1535)$ is excited/deexited by the exchange of δ–, σ–, π–, η–, ρ– and ω–mesons, while it is deexcited/excited by the produced η.

Total Cross Section

Using relativistic normalisations for spinors, creation and annihilation operators it is straight forward to write down the expression for the total cross section of $pp \to pp\eta$ ($P_i = p_1 + p_2$, $s = P_i^2$, $\mathcal{F}(s) = 2\sqrt{\lambda(s, m_p^2, m_p^2)}$, $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$):

$$\sigma_{pp\to pp\eta}(s) = \frac{1}{2!}\frac{1}{\mathcal{F}(s)}\frac{1}{(2\pi)^5}\int \frac{d^3p_{1'}}{2\,\omega_p(|\vec{p}_{1'}|)}\,\frac{d^3p_{2'}}{2\,\omega_p(|\vec{p}_{2'}|)}\,\frac{d^3p_{3'}}{2\,\omega_\eta(|\vec{p}_{3'}|)}\;\delta^4(p_{1'} + p_{2'} + p_{3'} - P_i)\;\overline{|T_{fi}|^2} \tag{1}$$

The combinatorial factor of $1/2!$ is due to the two outgoing identical protons. For the relativistic description of $pp \to pp\eta$ the following 5 independent invariants are chosen:

$$s = (p_1 + p_2)^2\ ,\ s_1 = (p_{1'} + p_{2'})^2\ ,\ s_2 = (p_{2'} + p_{3'})^2\ ,\ t_1 = (p_1 - p_{1'})^2\ ,\ t_2 = (p_2 - p_{3'})^2 \tag{2}$$

*Email: kleefeld@theorie3.physik.uni-erlangen.de / Publication No. FAU-TP3-99/7

(On–Shell) Watson-Migdal-Approach to ISI and FSI

In order to get a *quantitative* understanding of the ISI and FSI it is crucial to recall the steps which lead to that what the people nowadays call"Watson-Migdal approximation" and what is wrongly implemented in nearly all threshold meson production calculations by now: The system of colliding and produced particles is described by an overall Hamilton operator $H = K + V = K + W + U$, while K is the kinetic energy operator, W is the particle number nonconserving short range interaction potential and U is the particle number conserving interaction potential of long range. For convenience additionally one can define the long range Hamilton operator $h = K + U$. The corresponding eigenstates to the operators H, h and K fulfil the following Schrödinger equations:

$$(E - H)|\psi_\alpha^\pm >= 0 \quad , \quad (E - h)|\chi_\alpha^\pm >= 0 \quad , \quad (E_n - K)|\varphi_n^\pm >= 0 \tag{3}$$

The corresponding Lippmann–Schwinger equations are:

$$\begin{aligned} |\psi_\alpha^\pm > &= |\varphi_\alpha > + \frac{1}{E - H \pm i\varepsilon}\, V\, |\varphi_\alpha > = |\chi_\alpha^\pm > + \frac{1}{E - H \pm i\varepsilon}\, W\, |\chi_\alpha^\pm > \\ |\chi_\alpha^\pm > &= |\varphi_\alpha > + \frac{1}{E - h \pm i\varepsilon}\, U\, |\varphi_\alpha > = |\varphi_\alpha > + \frac{1}{E - K \pm i\varepsilon}\, T_{el}^\pm(E,K)\, |\varphi_\alpha > \end{aligned} \tag{4}$$

Defining the free propagator $G_0^\pm(E,K) = (E - K \pm i\varepsilon)^{-1}$ and inserting complete sets of free asymptotic states the T-matrix T_{fi} in Eq. (1) can be separated in a short and long ranged part via ($T_{\beta\alpha} = < \chi_\beta^-|W|\psi_\alpha^+ > = < \psi_\beta^-|W|\chi_\alpha^+ >$):

$$\begin{aligned} T_{\beta\alpha} \simeq < \chi_\beta^-|W|\chi_\alpha^+ > &= < \varphi_\beta|(1 + \sum_n T_{el}^+(E,K)|\varphi_n > G_0^+(E,E_n) < \varphi_n|)W \\ &\quad (1 + \sum_m |\varphi_m > G_0^+(E,E_m) < \varphi_m|T_{el}^+(E,K))|\varphi_\alpha > \end{aligned} \tag{5}$$

As the in- and outgoing particles in $pp \to pp\eta$ are nearly on-shell one can approximate the free propagator $G_0^\pm(E,K)$ in the following way:

$$G_0^\pm(E,K) = \frac{1}{E - K \pm i\varepsilon} = P\frac{1}{E-K} \mp i\pi\delta(E-K) \approx \mp i\pi\delta(E-K) \tag{6}$$

At this point it is useful to introduce the following relative lab wavenumbers for each pair of particles in the initial and final state ($s_3 = (p_{3'} + p_{1'})^2 = s - s_1 - s_2 + 2m_p^2 + m_\eta^2$):

$$k = \frac{\sqrt{\lambda(s, m_p^2, m_p^2)}}{2(m_p + m_p)} \; , \; \kappa = \frac{\sqrt{\lambda(s_1, m_p^2, m_p^2)}}{2(m_p + m_p)} \; , \; \kappa_1 = \frac{\sqrt{\lambda(s_3, m_p^2, m_\eta^2)}}{2(m_p + m_\eta)} \; , \; \kappa_2 = \frac{\sqrt{\lambda(s_2, m_p^2, m_\eta^2)}}{2(m_p + m_\eta)} \tag{7}$$

and the corresponding wavevectors $\vec{k}$, $\vec{\kappa}$, $\vec{\kappa}_1$, $\vec{\kappa}_2$. Consider now the case, when the only two particles taking part in the ISI and FSI are the two in– and outgoing protons. In this case the subsystem of the two protons interacting via the long range potental U with phaseshifts δ_ℓ (ℓ = orbital angular momentum) can be treated within the framework of standard nonrelativistic scattering theory. By application of the on-shell approximation of Eq. (6) the T-matrix in Eq. (5) can be transformed to ($\vec{k}\,' = \vec{\kappa}$, $k' = \kappa$):

$$\begin{aligned} T_{fi} \simeq T(\vec{k}\,', \ell'; \vec{k}, \ell) &= \left(1 + ie^{i\delta_{\ell'}(k')} \sin\delta_{\ell'}(k')\right) < \vec{k}\,', \ell'|W|\vec{k}, \ell > \left(1 + ie^{i\delta_\ell(k)} \sin\delta_\ell(k)\right) \\ &= \frac{k' \cot\delta_{\ell'}(k')}{k'(\cot\delta_{\ell'}(k') - i)} < \vec{k}\,', \ell'|W|\vec{k}, \ell > \frac{k\cot\delta_\ell(k)}{k(\cot\delta_\ell(k) - i)} \\ &= \frac{\mathrm{Re} f_{\ell'}(k')}{f_{\ell'}(k')} < \vec{k}\,', \ell'|W|\vec{k}, \ell > \frac{\mathrm{Re} f_\ell(k)}{f_\ell(k)} = T(\mathrm{FSI})\; \bar{T}_{fi}\; T(\mathrm{ISI}) \end{aligned} \tag{8}$$

Here the Jost functions $f_\ell(k)$ were introduced by $e^{i\delta_\ell(k)} \sin\delta_\ell(k) = -(\mathrm{Im} f_\ell(k))/f_\ell(k)$. It is worth to mention that the ISI– and FSI–factors of Eq. (8) have the correct limit for vanishing ISI or FSI. If there is no ISI or FSI, the phaseshifts will vanish and the corresponding factors will go to 1. The effect of the Watson-Migdal approach is a factorization of T_{fi} into a short ranged T-matrix $\bar{T}_{fi}$ and ISI- and FSI-factors T(ISI) and T(FSI).

The the factor $T(\mathrm{ISI})$ is only a function of s, while the short ranged T-matrix $\bar{T}_{fi}$ close to threshold shows up to be a slowly varying function of the phasespace integration variables and therefore can be set to its threshold value $\bar{T}_{fi}^{\mathrm{thr}}$. Using this observation both factors can be drawn in front of the phasespace integral in Eq. (1) with the result:

$$\begin{aligned}
\sigma_{pp\to pp\eta}(s) \quad &\simeq \\
&\simeq \quad \frac{\left|\bar{T}_{fi}^{\mathrm{thr}}\, T(\mathrm{ISI})\right|^2}{2!\mathcal{F}(s)(2\pi)^5} \int \frac{d^3p_{1'}}{2\,\omega_p(|\vec{p}_{1'}|)}\, \frac{d^3p_{2'}}{2\,\omega_p(|\vec{p}_{2'}|)}\, \frac{d^3p_{3'}}{2\,\omega_\eta(|\vec{p}_{3'}|)}\; \delta^4(p_{1'}+p_{2'}+p_{3'}-P_i)\; |T(\mathrm{FSI})|^2 \\
&= \quad \frac{\left|\bar{T}_{fi}^{\mathrm{thr}}\, T(\mathrm{ISI})\right|^2}{2!\mathcal{F}(s)(2\pi)^5}\, R_3^{FSI}(s) \qquad (9)
\end{aligned}$$

In the last line of Eq. (9) the final state interaction modified phasespace integral $R_3^{FSI}(s)$ for the three body final state $pp\eta$ has been defined. To expand the energy dependence of the total cross section at threshold one usually expresses the cross section in terms of the dimesionless variable $\eta = \sqrt{\lambda(s, m_\eta^2, s_1^{\min})/(4s\, m_\eta^2)}$ instead of the square of the cm-energy s. The quantity η being the maximum momentum of the produced η-meson in the cm-frame vanishes at threshold. It is easy to derive $s_1^{\min} = (2m_p)^2$. Introducing $T_{\mathrm{lab}} = (s - 4m_p^2)/(2m_p)$ and $\mu = m_p/m_\eta$ the phaseshifts of the long range potential between the incoming two protons can be expanded at threshold:

$$\delta(T_{\mathrm{lab}}) = \underbrace{\delta(T_{\mathrm{lab}}^{\mathrm{thr}})}_{=:\,\delta^{(0)}} + (T_{\mathrm{lab}} - T_{\mathrm{lab}}^{\mathrm{thr}})\delta'(T_{\mathrm{lab}}^{\mathrm{thr}}) + \ldots = \delta^{(0)} + \underbrace{m_\eta \left(1+\frac{1}{2\mu}\right)^2 \delta'(T_{\mathrm{lab}}^{\mathrm{thr}})}_{=:\,\delta^{(2)}}\, \eta^2 + O(\eta^4) \qquad (10)$$

In terms of these expansion coefficents the factor $T(\mathrm{ISI})$ can be expanded at threshold:

$$T(\mathrm{ISI}) \;=\; 1 + i\, e^{i\delta(k)} \sin\delta(k) \;=\; 1 + i e^{i\delta^{(0)}} \sin\delta^{(0)} + i\, e^{2i\delta^{(0)}} \delta^{(2)} \eta^2 + O(\eta^4) \qquad (11)$$

At the moment there is no clear knowledge of the values of $\delta^{(0)}$ and $\delta^{(2)}$ at the production threshold of $pp \to pp\eta$, but as the incoming relative proton momentum is very large the interaction time is very short, so that one might assume $\delta^{(0)}$ and $\delta^{(2)}$ to be zero which yields $T(\mathrm{ISI}) \approx 1$, while the VPI-values $\delta^{(0)} \approx -60^o$, $\delta^{(2)} \approx 0$ lead to $|T(\mathrm{ISI})| \approx 1/2$.
For the complete determination of $T(\mathrm{FSI})$ one obviously has to solve the Faddeev equations for the outgoing $pp\eta$–system. As this is at the moment out of the scope of this work only the leading terms of the Fadeev expansion are taken into account:

$$T(\mathrm{FSI}) \;\approx\; \frac{\kappa \cot\delta_{1'2'}(\kappa)}{\kappa(\cot\delta_{1'2'}(\kappa) - i)} \;+\; \frac{\kappa_1 \cot\delta_{3'1'}(\kappa_1)}{\kappa_1(\cot\delta_{3'1'}(\kappa_1) - i)} \;+\; \frac{\kappa_2 \cot\delta_{2'3'}(\kappa_2)}{\kappa_2(\cot\delta_{2'3'}(\kappa_2) - i)} \;-\; 2 \qquad (12)$$

The various terms in Eq. (12) which denote the FSI between each pair of particles in the final state can be expanded by effective range expansions. For the outgoing pp–system the s-wave nuclear effective range expansion is given by:

$$\kappa \cot\delta_{1'2'}(\kappa) = -\frac{1}{a} + \frac{r}{2}\kappa^2 + O(\kappa^4) \quad \text{with} \quad a \approx -17.1\,\mathrm{fm}\;,\; r \approx 0\ldots 2.84\,\mathrm{fm} \qquad (13)$$

Taking into account only pp-FSI using the shape independent effective range expansion Eq. (13) the FSI-modified phasespace integral can be expanded at threshold ($\bar{a} = a\, m_\eta$):

$$\begin{aligned}
R_3^{FSI}(s) \quad &\simeq \quad \int \frac{d^3p_{1'}}{2\,\omega_p(|\vec{p}_{1'}|)}\, \frac{d^3p_{2'}}{2\,\omega_p(|\vec{p}_{2'}|)}\, \frac{d^3p_{3'}}{2\,\omega_\eta(|\vec{p}_{3'}|)}\; \delta^4(P_f - P_i) \left| \frac{-\frac{1}{a} + \frac{r}{2}\kappa^2}{-\frac{1}{a} + \frac{r}{2}\kappa^2 - i\kappa} \right|^2 \\
&= \quad \frac{\pi^3}{4}\, m_\eta^2\, \frac{\sqrt{1+2\mu}}{2\mu} \left\{ \frac{\eta^4}{2^2} - \frac{\eta^6}{2^6}[4(2\mu)^{-2} + 7 + 2\bar{a}^2 + 2\bar{a}^2(2\mu)] + O(\eta^8) \right\} \qquad (14)
\end{aligned}$$

It is interesting to observe that even the η^6-term is independent of the effective range r.

Relativistic Meson-Exchange-Amplitudes

For the relativistic meson-exchange model the following compact expression for the ${}^3P_0 \to {}^1S_0 s$ threshold transition amplitude $\bar{T}_{fi}^{\rm thr}$ of $pp \to pp\eta$ is obtained [1,2] ($m_N \simeq m_p$):

$$\begin{aligned}
\bar{T}_{fi}^{\rm thr} &= 2\, m_N \sqrt{m_\eta\,(m_\eta + 4\, m_N)}\; [\,(\,X_\delta + X_\sigma\,)\,(m_N + m_\eta) - (\,X_\pi + X_\eta\,)\,(m_N - m_\eta) + \\
&+ (\,Y_\delta + Y_\sigma - Y_\pi - Y_\eta\,)\,(m_N + m_\eta) + M_\delta + M_\sigma - M_\pi - M_\eta\,] - \\
&- X_\rho\; m_N\; [\,4\,(m_\eta - 2\, m_N) + K_\rho\,(5 m_\eta - 4\, m_N)\,] + \\
&+ [\,Y_\rho\,(m_N + m_\eta) + \tilde{M}_\rho\,]\,[\,K_\rho\,(m_\eta - 4\, m_N) - 8\, m_N\,] - \\
&- X_\omega\; m_N\; [\,4\,(m_\eta - 2\, m_N) + K_\omega\,(5 m_\eta - 4\, m_N)\,] + \\
&+ [\,Y_\omega\,(m_N + m_\eta) + \tilde{M}_\omega\,]\,[\,K_\omega\,(m_\eta - 4\, m_N) - 8\, m_N\,] \qquad (K_\rho \approx 6.1, K_\omega \approx 0) \quad (15)
\end{aligned}$$

Here I used the following abbreviations ($\phi \in \{\delta, \sigma, \pi, \eta, \rho, \omega\}$) ($M_{S_{11}} := m_{S_{11}} - i\,\Gamma_{S_{11}}/2$) ($D_\phi(q^2) := (q^2 - m_\phi^2)^{-1}$) ($q^2 := -m_p m_\eta$, $p^2 := m_p\,(m_p - 2m_\eta)$, $P^2 := (m_p + m_\eta)^2$):

$$\begin{aligned}
X_\phi &:= D_\phi(q^2)\; g_{\phi NN}(q^2)\; D_{S_{11}}(p^2)\; g^*_{\phi N S_{11}}(q^2)\; g_{\eta N S_{11}}(m_\eta^2) \\
Y_\phi &:= D_\phi(q^2)\; g_{\phi NN}(q^2)\; D^R_{S_{11}}(P^2)\; g^*_{\eta N S^R_{11}}(m_\eta^2)\; g_{\phi N S^L_{11}}(q^2) \\
M_\phi &:= X_\phi\, m_{S_{11}} + Y_\phi\, M_{S_{11}} \quad , \quad \tilde{M}_\phi := -X_\phi\, m_{S_{11}} + Y_\phi\, M_{S_{11}} \\
D^R_{S_{11}}(P^2) &:= (P^2 - M^2_{S_{11}})^{-1} \; , \quad D_{S_{11}}(p^2) := (p^2 - m^2_{S_{11}})^{-1} \qquad (16)
\end{aligned}$$

Coupling Constants and Vertexfunctions

The meson–nucleon–$S_{11}(1535)$ couplings are in general complex and show a strong non-trivial momentum sensitivity. For that reason it is not enough to evaluate Eq. (16) applying the commonly used on-shell coupling constants combined with standard monopole or dipole formfactors. Hence a model has been developed to estimate the real and imaginary parts of the couplings [1,3]. The real parts of the couplings are derived within the framework of a nonrelativistic quark model, in which the ground state nucleon and the $S_{11}(1535)$ wavefunctions are described by harmonic oscillator solutions. The radial wavefunctions R_p, $R^{(23)}_{S_{11}}$ and $R^{(1,23)}_{S_{11}}$ of the ground state nucleon and the $S_{11}(1535)$ are determined by one unique range parameter b (Here ρ_{23}, $\rho_{1,23}$ are 3–quark–Jacobian coordinates!):

$$R_p \propto e^{-b^2(\rho^2_{23} + \rho^2_{1,23})/2}, \; R^{(23)}_{S_{11}} \propto b\,\rho_{23}\, e^{-b^2(\rho^2_{23} + \rho^2_{1,23})/2}, \; R^{(1,23)}_{S_{11}} \propto b\,\rho_{1,23}\, e^{-b^2(\rho^2_{23} + \rho^2_{1,23})/2} \quad (17)$$

The imaginary parts of the couplings are calculated close the threshold from relevant lowest order meson loop corrections to the bare couplings.

FIRST RESULTS

Taking into account only π–, η–, ρ– and ω–exchange we observe by reproducing the experimental total cross section of $pp \to pp\eta$ that the following mixing parameters are favourable: $\theta \approx -5^o, b^{-1} \approx 0.5$ fm. Because of the uncertainties in the bare meson-nucleon-$S_{11}(1535)$ couplings and the still improvable treatment of ISI and FSI these numbers are by now preliminary. The model gives for the first time a quantitative prediction of the relative phases between the π–, η–, ρ– and ω–exchange amplitudes. An extension of the effective range formalism for ISI and FSI to Coulomb-interactions is on the way.

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