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PROTONINDUCED PION-, ETA- AND KAON-PRODUCTION AT THRESHOLD [a]

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We present some details of our recent theoretical investigations on the total cross section of the reactions $pp \to pp\,\pi^0$, $pp \to pp\,\eta$ and $pp \to p\Lambda K^+$ at threshold. Applying standard nonrelativistic perturbation theory within the framework of meson exchange models (MEC) to all of the investigated processes we observe that theory is still far–off from a quantitative understanding of the existing data, although the qualitative behaviour of the threshold cross sections can be reproduced. The often quoted η^2–dependence of the $pp \to pp\,\pi^0$ cross section at threshold is due to a calculational mistake of Koltun and Reitan (1966) and not based on theoretical grounds, where threebody phasespace behaviour would yield η^4, which can be affected by final state and short range effects. To respect the high momentum transfers for $pp \to pp\,\eta$ and $pp \to p\Lambda K^+$ relativistic models were developed. Hereby the η–production still is based on MEC, while the K^+ production was investigated in the quark-gluon-picture based on the Bethe–Salpeter formalism.

1 Final state interactions in the $pp\,\pi^0$ and $pp\,\eta$–system

The energy dependence of the cross section of $pp \to pp\,\phi$ ($\phi \in \pi^0, \eta$) at threshold is determined by final state modified phasespace integrals [1]. A simple example is the shape independent effective range expansion in the pp–final state ($s_1 := (p_1' + p_2')^2$, $\kappa := \sqrt{\lambda(s_1, m_p^2, m_p^2)}/(4m_p)$, $s = P^2$):

$$R_3^{\,\mathrm{FSI}}(s\ ;\ m_p^2, m_p^2, m_\phi^2) \approx \int \frac{d^3p_{1'}}{2\,\omega_p(|\vec{p}_{1'}|)}\ \frac{d^3p_{2'}}{2\,\omega_p(|\vec{p}_{2'}|)}\ \frac{d^3k_\phi}{2\,\omega_\phi(|\vec{k}_\phi|)}$$
$$\cdot\quad \delta^4(p_{1'} + p_{2'} + k_\phi - P)\left| -\frac{\mathrm{N}_\phi}{a_{pp}} \frac{1}{(-1/a_{pp}\ +\ r_{pp}\,\kappa^2/2 - i\kappa)} \right|^2 \ \alpha\ \ \eta_\phi^4 \quad (1)$$

The dimensionless normalization N_ϕ is an open question in nearly all theoretical models in the literature which prevents them from being quantitative.

2 The reaction $pp \to pp\,\pi^0$ at threshold

Applying a nonrelativistic meson exchange model [1,2] to $pp \to pp\pi^0$ at threshold we are able to describe the total cross section qualitatively upto $\eta_\pi \approx 0.3$.

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The model contains contributions from IA, offshell rescattering, heavy meson exchanges, resonant S– and P–waves, recoil currents and $\pi\pi$–box–diagrams.

3 The reaction $pp \to pp\,\eta$ at threshold

The total cross section of $pp \to pp\,\eta$ at threshold has been calculated within a nonrelativistic and a relativistic meson–exchange model [1,2] (exciting the $S_{11}^-(1535)$) which is based on the following interaction Lagrangian:

$$\begin{aligned}
\mathcal{L}_{int}(x) = & - g_{\delta NN}\bar{N}\vec{\tau}\cdot\vec{\phi}_\delta\, N - g_{\sigma NN}\bar{N}\phi_\sigma\, N - i\, g_{\pi NN}\bar{N}\gamma_5\,\vec{\tau}\cdot\vec{\phi}_\pi\, N - \\
- & \quad i\, g_{\eta NN}\bar{N}\gamma_5\,\phi_\eta\, N - g_{\rho NN}\,\bar{N}\,\vec{\tau}\cdot[\gamma_\mu\,\vec{\phi}_\rho^{\,\mu} + \frac{K_\rho}{2m_N}\,\sigma_{\mu\nu}\,\partial^\mu\vec{\phi}_\rho^{\,\nu}]\,N - \\
- & \quad g_{\omega NN}\,\bar{N}\,[\gamma_\mu\,\phi_\omega^\mu + \frac{K_\omega}{2m_N}\,\sigma_{\mu\nu}\,\partial^\mu\phi_\omega^\nu]\,N + \Big[\, i\, g_{\delta NS_{11}}\;\bar{N}_{S_{11}}\gamma_5\,\vec{\tau}\cdot\vec{\phi}_\delta\;N\; + \\
+ & \quad i\, g_{\sigma NS_{11}}\;\bar{N}_{S_{11}}\gamma_5\,\phi_\sigma\;N - g_{\pi NS_{11}}\;\bar{N}_{S_{11}}\vec{\tau}\cdot\vec{\phi}_\pi\;N - g_{\eta NS_{11}}\;\bar{N}_{S_{11}}\phi_\eta\;N\; + \\
+ & \quad i\, g_{\rho NS_{11}}\,\bar{N}_{S_{11}}\gamma_\mu\gamma_5\,\vec{\tau}\cdot\vec{\phi}_\rho^{\,\mu}N + i\, g_{\omega NS_{11}}\,\bar{N}_{S_{11}}\gamma_\mu\gamma_5\,\phi_\omega^\mu N + h.c.\Big] \qquad (2)
\end{aligned}$$

Following Watson and Migdal we multiplicatively separate the long-ranged final state interactions from the T–matrix. The short ranged (approximately constant) production amplitude is set to its threshold value. The resulting cross section of the relativistic model is ($m_N \simeq (m_p + m_n)/2$):

$$\begin{aligned}
\sigma_{pp\to pp\,\eta}(s) \quad \simeq \quad & \frac{1}{2\,!}\,\frac{1}{(2\pi)^5}\;\frac{R_3^{\rm FSI}(s\;;\;m_p^2, m_p^2, m_\eta^2)}{2\,\sqrt{\lambda(s\;;\;m_p^2, m_p^2)}}\;\; m_\eta\;(m_\eta + 4\,m_{_N})\;\cdot \\
\cdot \quad & \Big|\, 2\,m_{_N}\;[\,(\,X_\delta + X_\sigma\,)\,(m_{_N} + m_\eta) - (\,X_\pi + X_\eta\,)\,(m_{_N} - m_\eta)\;+ \\
& + (\,Y_\delta + Y_\sigma - Y_\pi - Y_\eta\,)\,(m_{_N} + m_\eta) + M_\delta + M_\sigma - M_\pi - M_\eta\,] - \\
& - X_\rho\;m_{_N}\;[\,4\,(m_\eta - 2\,m_{_N}) + K_\rho\,(5m_\eta - 4\,m_{_N})\,]\;+ \\
& + [\,Y_\rho\,(m_{_N} + m_\eta) + \tilde{M}_\rho\,]\,[\,K_\rho\,(m_\eta - 4\,m_{_N}) - 8\,m_{_N}\,] - \\
& - X_\omega\;m_{_N}\;\;[\,4\,(m_\eta - 2\,m_{_N}) + K_\omega\,(5m_\eta - 4\,m_{_N})\,]\;+ \\
& + [\,Y_\omega\,(m_{_N} + m_\eta) + \tilde{M}_\omega\,]\,[\,K_\omega\,(m_\eta - 4\,m_{_N}) - 8\,m_{_N}\,]\;\Big|^2 \qquad (3)
\end{aligned}$$

with [3] ($\phi \in \{\delta, \sigma, \pi, \eta, \rho, \omega\}$) ($M_{S_{11}} := m_{S_{11}} - i\,\Gamma_{S_{11}}/2$)($D_\phi(q^2) := (q^2 - m_\phi^2)^{-1}$) ($q^2 := -m_p m_\eta$, $p^2 := m_p\;(m_p - 2m_\eta)$, $P^2 := (m_p + m_\eta)^2$):

$$\begin{aligned}
X_\phi &:= D_\phi(q^2)\; g_{\,\phi NN}(q^2)\; D_{S_{11}}(p^2)\; g^*_{\phi NS_{11}}(q^2)\; g_{\eta NS_{11}}(m_\eta^2) \\
Y_\phi &:= D_\phi(q^2)\; g_{\,\phi NN}(q^2)\; D^R_{S_{11}}(P^2)\; g^*_{\eta NS^R_{11}}(m_\eta^2)\; g_{\Phi NS^L_{11}}(q^2)
\end{aligned}$$

$$M_\phi := X_\phi\, m_{S_{11}} + Y_\phi\, M_{S_{11}} \quad , \quad \tilde{M}_\phi := -X_\phi\, m_{S_{11}} + Y_\phi\, M_{S_{11}}$$
$$D^R_{S_{11}}(P^2) := (P^2 - M^2_{S_{11}})^{-1} \ , \quad D_{S_{11}}(p^2) := (p^2 - m^2_{S_{11}})^{-1} \qquad (4)$$

4 The reaction $pp \to p\Lambda K^+$ at threshold

To open the door for future investigations we sketch out some details of our convariant quark–gluon Bethe–Salpeter (BS) model to describe $pp \to p\Lambda K^+$ at threshold[4]. The total cross section of $pp \to p\Lambda K^+$ is given by:

$$\sigma_{pp\to p\Lambda K}(s) \quad =$$
$$= \quad \frac{1}{2\,\sqrt{\lambda(s; m_p^2, m_p^2)}} \int \frac{d^{\,3}p_{1'}}{(2\pi)^3\, 2\,\omega_p(|\vec{p}_{1'}|)} \frac{d^{\,3}p_{2'}}{(2\pi)^3\, 2\,\omega_\Lambda(|\vec{p}_{2'}|)} \frac{d^{\,3}k}{(2\pi)^3\, 2\,\omega_K(|\vec{k}|)}$$
$$(2\pi)^4\, \delta^4(p_{1'} + p_{2'} + k - p_1 - p_2)\, \frac{1}{4} \sum_{s_1, s_2} \sum_{s_{1'}, s_{2'}} |\, T_{fi}\,|^2 \qquad (5)$$

$$i\,T_{fi} \quad = \quad < p\,(\vec{p}_1\,', s_1')\, \Lambda^0(\vec{p}_2\,', s_2')\, K^+(\vec{k})|\, T \left[\frac{i}{1!}\, \hat{\mathcal{L}}_{int}(0) + \frac{i^2}{2!}\, \hat{\mathcal{L}}_{int}(0)\, \hat{\mathcal{S}}_{int} + \right.$$
$$+ \quad \left. \frac{i^3}{3!}\, \hat{\mathcal{L}}_{int}(0)\, \hat{\mathcal{S}}_{int}\, \hat{\mathcal{S}}_{int} + \ldots \right] |p\,(\vec{p}_1, s_1)\, p(\vec{p}_2, s_2)\, >_c \qquad (6)$$

$$\hat{\mathcal{L}}_{int}(x) \simeq \sqrt{4\pi\alpha_s}\ \ \bar{\psi}(x)\, \frac{\lambda_a}{2}\ \not{A}_a(x)\, \psi(x)\ ,\ \alpha_s(Q^2) \simeq 4\pi(\beta_0)^{-1} / \ln\left(1 + \frac{Q^2}{\Lambda^2_{QCD}}\right) \qquad (7)$$

($\hat{\mathcal{S}}_{int} := \int d^{\,4}x\ \hat{\mathcal{L}}_{int}(x)$). The in– and outgoing protons are considered to be bare three–quark objects dressed by scalar $q\bar{q}$–pairs. In our model we use properly normalised momentum space quark and antiquark creation (b^+, d^+) and annihilation (b, d) operators $(\{b(\vec{p}, \alpha), b^+(\vec{p}\,', \alpha')\} = \delta_{\alpha\alpha'}\ (2\pi)^3\ 2\,\omega(|\vec{p}\,|)\ \delta^3(\vec{p} - \vec{p}\,'),\ \ldots)$ and Dirac spinors $(\bar{u}(\vec{p}, s)\, u(\vec{p}, s') = 2m\ \delta_{s\,s'},\ \bar{v}(\vec{p}, s)\ v(\vec{p}, s') = -2m\ \delta_{ss'})$ to express the asymptotic in– and outgoing proton state vectors and the outgoing Kaon state vector (upto renormalisation constants) in terms of corresponding BS–amplitudes (s_i, t_i, c_i denote spin, flavour and colour):

$$|K^+(P)> \quad \simeq \quad - \sum_{s_1, s_2} \sum_{t_1, t_2} \sum_{c_1, c_2} \int \frac{d^{\,3}p_1}{(2\pi)^3\, 2m_1}\, \frac{d^{\,3}p_2}{(2\pi)^3\, 2m_2}\ \int d^{\,3}x_1\, d^{\,3}x_2$$
$$\cdot \quad \exp(-i\,(\vec{p}_1 \cdot \vec{x}_1 + \vec{p}_2 \cdot \vec{x}_2))\ \cdot$$
$$\cdot \quad \bar{u}^{\,(1)}(\vec{p}_1, s_1, t_1, c_1) \quad <0|\, T\left(\psi^{\,(1)}(x_1)\, \bar{\psi}^{\,(2)}(x_2)\right)\ |P, K^+> \cdot$$
$$\cdot \quad v^{\,(2)}(\vec{p}_2, s_2, t_2, c_2)\ d^+(\vec{p}_2, s_2, t_2, c_2)\ b^+(\vec{p}_1, s_1, t_1, c_1)\ |0> \Big|_{x_1^0\,=\,x_2^0\,=\,0} \qquad (8)$$

$$
\begin{aligned}
|p^+(P)> \;=\; & \sum_{s_1,s_2,s_3}\sum_{t_1,t_2,t_3}\sum_{c_1,c_2,c_3}\int \frac{d^3p_1}{(2\pi)^3\,2\,m_1}\,\frac{d^3p_2}{(2\pi)^3\,2\,m_2}\,\frac{d^3p_3}{(2\pi)^3\,2\,m_3} \\
& \int d^3x_1\,d^3x_2\,d^3x_3\;\exp[-i\,(\vec{p}_1\cdot\vec{x}_1+\vec{p}_2\cdot\vec{x}_2+\vec{p}_3\cdot\vec{x}_3)]\;\cdot \\
& \Big\{\;\bar{u}^{(1)}(\vec{p}_1,s_1,t_1,c_1)\;\bar{u}^{(2)}(\vec{p}_2,s_2,t_2,c_2)\;\bar{u}^{(3)}(\vec{p}_3,s_3,t_3,c_3) \\
& <0|\,T\left(\psi^{(1)}(x_1)\,\psi^{(2)}(x_2)\,\psi^{(3)}(x_3)\right)\,|P,p^+> \\
& b^+(\vec{p}_3,s_3,t_3,c_3)\;b^+(\vec{p}_2,s_2,t_2,c_2)\;b^+(\vec{p}_1,s_1,t_1,c_1)\;|0> \;-\; \sum_{s_4,s_5}\sum_{t_4,t_5}\sum_{c_4,c_5} \\
& \int \frac{d^3p_4}{(2\,\pi)^3\,2\,m_4}\,\frac{d^3p_5}{(2\,\pi)^3\,2\,m_5}\int d^3x_4\,d^3x_5\;\exp[-i\,(\vec{p}_4\cdot\vec{x}_4+\vec{p}_5\cdot\vec{x}_5)]\;\cdot \\
& \bar{u}^{(1)}(\vec{p}_1,s_1,t_1,c_1)\;\bar{u}^{(2)}(\vec{p}_2,s_2,t_2,c_2)\;\bar{u}^{(3)}(\vec{p}_3,s_3,t_3,c_3)\;\bar{u}^{(4)}(\vec{p}_4,s_4,t_4,c_4) \\
& \quad <0|\,T\left(\psi^{(1)}(x_1)\,\psi^{(2)}(x_2)\,\psi^{(3)}(x_3)\,\psi^{(4)}(x_4)\,\bar{\psi}^{(5)}(x_5)\right)\,|P,p^+>\;\cdot \\
& \bar{v}^{(5)}(\vec{p}_5,s_5,t_5,c_5)\;d^+(\vec{p}_5,s_5,t_5,c_5)\;b^+(\vec{p}_4,s_4,t_4,c_4)\;b^+(\vec{p}_3,s_3,t_3,c_3) \\
& b^+(\vec{p}_2,s_2,t_2,c_2)\;b^+(\vec{p}_1,s_1,t_1,c_1)\;|0>+\ldots \quad \Big\}\Big|_{x_1^0\,=\,x_2^0\,=\,\ldots\,=\,0}
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
<K^+(\vec{P})|K^+(\vec{P}\,')> = \quad <P,K^+|P',K^+> \;&=\; (2\pi)^3\;2\,\omega_K(|\vec{P}\,|)\;\delta^3(\vec{P}-\vec{P}\,') \\
<p^+(\vec{P})|\,p^+(\vec{P}\,')> = \quad <P,p^+|P',p^+> \;&=\; (2\pi)^3\;\;2\,\omega_p(|\vec{P}\,|)\;\delta^3(\vec{P}-\vec{P}\,')
\end{aligned}
$$

A similar expression holds for the outgoing Λ–Baryon state $|\Lambda(P)>$. Both, the proton and the Λ are considered to be quark–diquark objects. Obviously the five quark BS–amplitude above is connected to the "strangeness content" of the proton. As for the baryons p^+ and Λ the BS–amplitude of the K^+ is calculated via a separable BS–kernel. The further evaluation of equ. (5) and (6) is performed via Wick's theorem. Final integrations are performed in lightcone variables. A similar approach to $pp \to pp\,\phi$ (quark–gluon picture) and $pp \to d\pi^+$ (nucleon–meson picture) at threshold is on the way.

References

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