Bethe–Salpeter–Approach to Relativistic Two–Fermion–Systems with Separable Nonstatic Interaction *

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Abstract

To study the characteristic features of relativistic bound systems, the Bethe-Salpeter equation (BSE) for two equal mass spin 1/2 particles (like the deuteron) is solved in the cm-frame for a separable and covariant interaction kernel. For that purpose the BSE is transformed to an eigenvalue problem which is diagonalized numerically. The Bethe–Salpeter amplitudes (BSAs) are obtained straightforwardly from the resulting eigenvectors. Only positive parity solutions of the eigenvalue problem are considered. To correlate the BSAs to standard quantum mechanical wavefunctions, the corresponding equal–time–wavefunctions (ETWs) are calculated. A decomposition of BSAs and ETWs in partial waves in angular momenta and parity is performed.

As a first application elastic electron–deuteron–scattering in the impulse approximation (IA) is considered following a similar route as in M.J. Zuilhof and J.A. Tjon 1980 [1]. The charge, magnetic and quadrupole formfactors $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ are obtained from three independent matrix elements of the deuteron current in the Breit–frame of elastic electron–deuteron–scattering. Additionally the formfactors $A(k^2)$ and $B(k^2)$ of the *Rosenbluth formula* are calculated.

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1 Introduction

The discussion of few and many body systems on a nuclear scale requires in most cases both a quantum mechanical and relativistic treatment. The appropriate, most probably the correct description of such systems is given by the BSE. Unfortunately the practical solution of the BSE for most particle dynamics is highly non–trivial and extremely complex. There are different ways out of this dilemma. One of them is to give up covariance and to use static interaction kernels in a Schrödinger or Dirac like description of the few or many particle problem excluding retardation effects in interaction.
The way we follow is to choose a covariant separable interaction kernel to satisfy covariance and to simplify the mathematical treatment of the BSE. As we are interested in bound systems we solve the homogenous BSE. Our test particle is the deuteron (mass M_d) as a weakly bound two–fermion–system of the two constituents proton and neutron (mass m, four–momentum p_1 and p_2).
The choice of our phenomenological interaction kernel results in a manageable, though still complex mathematical problem. Goal of this paper is the formulation and solution of the problem with a first simplistic application to elastic electron–deuteron–scattering. A more detailed discussion of the formalism and applications to selected systems will be presented elsewhere (e.g. the discussion of relativistic effects on bound state wavefuctions and formfactors or the investigation of antiparticle effects in few or many body systems at high energies).

2 Solution of the BSE in the rest frame of the deuteron

Starting point is the homogenous BSE ($P := p_1 + p_2\ ,\ \ q := \frac{1}{2}\,(p_1 - p_2)$):

$$S_{F2}^{-1}(P,q)\,\Psi(P,q) = -\frac{i}{(2\pi)^4}\int d^4k\, K(P,q\,;k)\,\Psi(P,k) \tag{1}$$

with

$$S_{F2}(P,q) = \Big[(\frac{1}{2}\,\not P + \not q - m + i\varepsilon)\otimes(\frac{1}{2}\,\not P - \not q - m + i\varepsilon)\Big]^{-1} \tag{2}$$

(to begin with we drop self–energy corrections to the nucleon mass).
The most general interaction kernel in terms of the various invariants of Dirac matrices is given as:

$$\begin{aligned} K(P\,,q\,;k) \;=\;\; & g^2\cdot(\;\; g_s\; f_s(P\,,q\,;k) \quad 1_4\otimes 1_4 \\ & \quad + \;g_v\; f_v(P\,,q\,;k) \quad \gamma_\mu\otimes\gamma^\mu \\ & \quad + \;g_t\; f_t(P\,,q\,;k) \quad \sigma_{\mu\nu}\otimes\sigma^{\mu\nu} \\ & \quad + \;g_p\; f_p(P\,,q\,;k) \quad \gamma_5\otimes\gamma^5 \\ & \quad + \;g_{pv}\, f_{pv}(P\,,q\,;k) \quad \gamma_5\gamma_\mu\otimes\gamma^5\gamma^\mu\;) \end{aligned} \tag{3}$$

For the practical calculation we use as interaction, respecting covariance, retardation effects and integrability of the Bethe–Salpeter–Equation (BSE), the following separable kernel ($q^2 = (q^0)^2 - \,|\vec{q}\,|^2$):

$$K(q\,;k) \;=\; g^2\,(g_s\,v_s(q)\,v_s(k)\;\Gamma_s + g_v\,v_v(q)\,v_v(k)\;\Gamma_v + g_p\,v_p(q)\,v_p(k)\;\Gamma_p) \qquad (4)$$

with

$$g_s = 1 \qquad \text{and} \qquad v_i(q) := \frac{\Lambda_i}{q^2 - \Lambda_i^2 + i\varepsilon} \qquad (i = s, v, p) \qquad (5)$$

and with the spin–structures:

$$\begin{aligned} \Gamma_s &= 1_4 \otimes 1_4 \\ \Gamma_v &\stackrel{!}{=} -\gamma^0 \otimes \gamma^0 + g_0 \sum_{i=1}^{3} \gamma^i \otimes \gamma^i \\ \Gamma_p &= \gamma^5 \otimes \gamma^5 \end{aligned} \qquad (6)$$

The interaction parameters Λ_s, Λ_v, Λ_p and the coupling constants g_v, g_p, g_0 are input parameters. The coupling constant g^2 will be calculated from the eigenvalue condition.

2.1 Formulation of the eigenvalue problem

Combination of the interaction kernel (4) and the homogenous BSE (1) yields:

$$S_{F2}^{-1}(P,q)\,\Psi(P,q) = -\frac{i}{(2\pi)^4}\int d^4k\,\left(g^2 \cdot \sum_j g_j\,v_j(q)\,v_j(k)\;\Gamma_j\right)\;\Psi(P,k) \qquad (j = s, v, p) \quad (7)$$

or after some transformations:

$$\Psi(P,q) = -\frac{ig^2}{(2\pi)^4} S_{F2}(P,q)\;\sum_j v_j(q)\,g_j\,\Gamma_j\;\int d^4k\;v_j(k)\;\Psi(P,k) \qquad (8)$$

Multiplication by $v_i(q)$ and integration over q leads to:

$$\int d^4q\;v_i(q)\;\Psi(P,q) = -\frac{ig^2}{(2\pi)^4}\int d^4q\;S_{F2}(P,q)\;\sum_j v_i(q)\,v_j(q)\,g_j\,\Gamma_j\;\int d^4k\;v_j(k)\;\Psi(P,k) \quad (9)$$

$$(i, j = s, v, p)$$

For convenience we introduce the following quantities:

$$X_i \;:=\; \int d^4q\;v_i(q)\;\Psi(P,q) \qquad (10)$$

$$A_{ij} \;:=\; -\frac{i}{(2\pi)^4}\int d^4q\;S_{F2}(P,q)\,v_i(q)\,v_j(q) \qquad (11)$$

(Note that X_i and A_{ij} are functions of the total four–momentum P !)

Using the definitions above the eigenvalue problem is obtained from (9) as:

$$\frac{1}{g^2} X_i = \sum_j A_{ij}\, g_j\, \Gamma_j\, X_j \qquad (i,j = s,v,p) \tag{12}$$

or equivalently ($\lambda := g^{-2}$):

$$\begin{pmatrix} (g_s A_{ss}\Gamma_s) & (g_v A_{sv}\Gamma_v) & (g_p A_{sp}\Gamma_p) \\ (g_s A_{vs}\Gamma_s) & (g_v A_{vv}\Gamma_v) & (g_p A_{vp}\Gamma_p) \\ (g_s A_{ps}\Gamma_s) & (g_v A_{pv}\Gamma_v) & (g_p A_{pp}\Gamma_p) \end{pmatrix} \begin{pmatrix} X_s \\ X_v \\ X_p \end{pmatrix} = \lambda \begin{pmatrix} X_s \\ X_v \\ X_p \end{pmatrix} \tag{13}$$

The (unnormalized) BSAs $\Psi(P,q)$ are calculated from the eigenvectors X_j and the corresponding eigenvalues g^2 from eq. (8) by:

$$\Psi(P,q) = -\frac{ig^2}{(2\pi)^4} S_{F2}(P,q) \sum_j v_j(q)\, g_j\, \Gamma_j\, X_j \tag{14}$$

Not all eigensolutions of the BSE are "physical" solutions; there are e.g. solutions with negative norm. Unphysical solutions of the BSE are sometimes called "Bethe–Salpeter ghosts" and are discussed e.g. in [2].
For our interaction kernel the normalization condition for the BSA reads:

$$\pi i \int d^4q\, \tilde{\Psi}_{B'}(P,q) \frac{\partial}{\partial P_\mu} [S_{F2}(P,q)]^{-1}\Big|_{P^2=M_d^2} \Psi_B(P,q) = P^\mu\, \delta_{B'B} \tag{15}$$

Here $\tilde{\Psi}$ is the adjoint BSA, B and B' are additional quantum numbers of the two–body bound states.

2.2 Calculation of A_{ij} in the rest frame of the two–particle system

For the numerical treatment of the eigenvalue problem (13) the matrix A_{ij} from eq. (11) has to be calculated. By the use of the definition of $v_i(q)$ in (5) and the rationalized version of the free two–fermion–propagator $S_{F2}(P,q)$ from (2):

$$S_{F2}(P,q) = \frac{(\frac{1}{2}\not{P} + \not{q} + m) \otimes (\frac{1}{2}\not{P} - \not{q} + m)}{((\frac{P}{2}+q)^2 - m^2 + i\varepsilon)((\frac{P}{2}-q)^2 - m^2 + i\varepsilon)} \tag{16}$$

the matrix A_{ij} can be rewritten as:

$$A_{ij} = -\frac{i}{(2\pi)^4} \int d^4q\, \frac{Z(P,q,m)}{N^{(ij)}(P,q,m)} \cdot \Lambda_i \Lambda_j \tag{17}$$

with

$$Z(P,q,m) := (\frac{1}{2}\not{P} + \not{q} + m) \otimes (\frac{1}{2}\not{P} - \not{q} + m) \tag{18}$$

$$N^{(ij)}(P,q,m) := ((\frac{P}{2}+q)^2 - m^2 + i\varepsilon)((\frac{P}{2}-q)^2 - m^2 + i\varepsilon)(q^2 - \Lambda_i^2 + i\varepsilon)(q^2 - \Lambda_j^2 + i\varepsilon) \tag{19}$$

Multiplying out the numerator $Z(P,q,m)$ leads to:

$$
\begin{aligned}
A_{ij} \stackrel{!}{=} \frac{\Lambda_i\Lambda_j}{(2\pi)^4} \Big\{ & (\tfrac{1}{4}P_\mu P_\nu\, I^{(ij)} - I^{(ij)}_{\mu\nu}) \quad (\gamma^\mu \otimes \gamma^\nu) & + \\
+ & \quad (\tfrac{m}{2}\, P_\mu\, I^{(ij)}) \quad ((\gamma^\mu \otimes 1_4) + (1_4 \otimes \gamma^\mu)) & + \\
+ & \quad (m^2\, I^{(ij)}) \quad (1_4 \otimes 1_4) & \Big\}
\end{aligned}
\tag{20}
$$

with the integrals:

$$
\begin{aligned}
I^{(ij)} &:= -i \int d^4q \, \frac{1}{N^{(ij)}(P,q,m)} \\
I^{(ij)}_\mu &:= -i \int d^4q \, \frac{q_\mu}{N^{(ij)}(P,q,m)} \quad \stackrel{!}{=} \quad 0 \\
I^{(ij)}_{\mu\nu} &:= -i \int d^4q \, \frac{q_\mu q_\nu}{N^{(ij)}(P,q,m)}
\end{aligned}
\tag{21}
$$

These integrations have been performed in the rest frame of the bound system; specifically for the deuteron $P^\mu = (M_d, \vec{0})$. The connection between M_d and E_B (E_B = binding energy of the deuteron) is:

$$
P^2 \quad = \quad (2\,m - E_B)^2 \quad = \quad M_d^2 \tag{22}
$$

2.3 Parity, angular momentum and the BSA

The 16×16–matrices $(g_j A_{ij} \Gamma_j)$ $(i,j = s,v,p)$ in (13) may be represented by 4×4–matrices with matrix elements being themselves 4×4–matrices, consisting of the 4×4–unity–matrix 1_4, the two particle spin operator σ and its square σ^2.
The two particle spin operator σ is defined as follows (σ^k with $k = 1,2,3$ are the Pauli matrices in the z–representation):

$$
\sigma \quad = \quad \sum_{k=1}^{3} \sigma^k \otimes \sigma^k \quad = \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Its eigenvalues λ_σ^S are:

$$
\begin{aligned}
\lambda_\sigma^0 &= -3 \quad \text{(singlet)} \\
\lambda_\sigma^1 &= 1 \quad \text{(triplet)}
\end{aligned}
$$

The corresponding orthonormal eigenvectors $\chi^S_{m_S}$ are for the singlet ($S = 0$) and for the triplet ($S = 1$) spin states:

$$\chi_0^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \qquad \chi_1^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \chi_0^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \qquad \chi_{-1}^1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The subspace can be separated from the eigenvalue problem (13) by the ansatz for the (16–component–)eigenvector X_j:

$$X_j = X_j^{(S)} = \begin{pmatrix} X_j^{++(S)} \\ X_j^{+-(S)} \\ X_j^{-+(S)} \\ X_j^{--(S)} \end{pmatrix} \chi_{m_S}^S \qquad (j = s, v, p) \tag{23}$$

The parity operator $\hat{\mathcal{P}}$ for the BSA is defined as:

$$\hat{\mathcal{P}} \quad := \quad (\gamma^0 \otimes \gamma^0)\ \hat{\mathcal{P}}^0 \tag{24}$$

with the so called "orbital parity operator" $\hat{\mathcal{P}}^0$:

$$\hat{\mathcal{P}}^0\ \Psi(P^0, \vec{P}; q^0, \vec{q}) \quad = \quad \Psi(P^0, -\vec{P}; q^0, -\vec{q}) \tag{25}$$

Finally, in our representation:

$$\gamma^0 \otimes \gamma^0 \quad = \quad \begin{pmatrix} 1_4 & 0 & 0 & 0 \\ 0 & -1_4 & 0 & 0 \\ 0 & 0 & -1_4 & 0 \\ 0 & 0 & 0 & 1_4 \end{pmatrix} \tag{26}$$

It is easy to show the effect of the parity operator $\hat{\mathcal{P}}$ on the BSA of (14) for $P^\mu = (M_d, \vec{0})$:

$$\hat{\mathcal{P}}\ \Psi(P,q) \stackrel{!}{=} -\frac{ig^2}{(2\pi)^4} S_{F2}(P,q) \sum_j v_j(q)\, g_j\, \Gamma_j\, (\gamma^0 \otimes \gamma^0)\, X_j \qquad (j = s, v, p) \tag{27}$$

i.e. the parity of the BSA only depends on the operation of $\gamma^0 \otimes \gamma^0$ on X_j. As the parity operator $\hat{\mathcal{P}}$ commutes with the interaction kernel $K(q\,;k)$ in (4), there exist eigensolutions of the BSA with positive and negative parity which can be selected (as one can easily see from (27) and (26)) with the following X_j $(j = s, v, p)$:

$$X_{+j}^{(S\,m_S)} := \begin{pmatrix} X_j^{++(S)} \\ 0 \\ 0 \\ X_j^{--(S)} \end{pmatrix} \chi_{m_S}^S \quad , \quad X_{-j}^{(S\,m_S)} := \begin{pmatrix} 0 \\ X_j^{+-(S)} \\ X_j^{-+(S)} \\ 0 \end{pmatrix} \chi_{m_S}^S \qquad (j = s, v, p) \tag{28}$$

For the deuteron only the states of positive parity (e.g. ${}^3S_1$, ${}^3D_1$, ...) (spectroscopic notation: ${}^{2S+1}L_J$) are observed. For that reason we only will consider eigensolutions of positive parity in the next few pages. For completeness we mention that in case of negative parity states the eigenvalue problem (13) can be solved analytically.

For a decomposition of the BSA into angular momenta we introduce the two fermion spherical harmonics $Y^M_{LSJ}(\Omega)$ which couple the spin angular momentum of a spin 1 particle (represented by the four–spinors $\chi^S_{m_S}$) to the orbital angular momentum (represented by the ordinary sperical harmonics $Y_{Lm_L}(\Omega)$):

$$Y^M_{LSJ}(\Omega) = \sum_{m_L, m_S} < LSm_L m_S | JM > Y_{Lm_L}(\Omega)\, \chi^S_{m_S} \tag{29}$$

with the orthonormality relation:

$$\int d\Omega\, Y^{M'\,\dagger}_{L'S'J'}(\Omega)\, Y^M_{LSJ}(\Omega) = \delta_{M'M}\, \delta_{L'L}\, \delta_{S'S}\, \delta_{J'J} \tag{30}$$

(The Clebsch–Gordan–coefficients follow the convention of Condon & Shortley [3]). The application of the projections:

$$\chi^J_M = \sqrt{4\pi}\, Y^M_{0JJ}(\Omega) \;\; (J=0,\; M=0 \text{ or } J=1,\; M=0,\pm 1) \tag{31}$$

$$\left(\frac{\vec\sigma\cdot\vec q}{|\vec q|} \otimes 1_2\right) \chi^0_0 = -\sqrt{4\pi}\, Y^0_{110}(\Omega) \tag{32}$$

$$\left(\frac{\vec\sigma\cdot\vec q}{|\vec q|} \otimes 1_2\right) \chi^1_M = \sqrt{\frac{4\pi}{3}}\, (Y^M_{101}(\Omega) - \sqrt{2}\, Y^M_{111}(\Omega)) \qquad (M=0,\pm 1) \tag{33}$$

$$\left(1_2 \otimes \frac{\vec\sigma\cdot\vec q}{|\vec q|}\right) \chi^0_0 = \sqrt{4\pi}\, Y^0_{110}(\Omega) \tag{34}$$

$$\left(1_2 \otimes \frac{\vec\sigma\cdot\vec q}{|\vec q|}\right) \chi^1_M = \sqrt{\frac{4\pi}{3}}\, (-Y^M_{101}(\Omega) - \sqrt{2}\, Y^M_{111}(\Omega)) \qquad (M=0,\pm 1) \tag{35}$$

$$\left(\frac{\vec\sigma\cdot\vec q}{|\vec q|} \otimes \frac{\vec\sigma\cdot\vec q}{|\vec q|}\right) \chi^0_0 = -\sqrt{4\pi}\, Y^0_{000}(\Omega) \tag{36}$$

$$\left(\frac{\vec\sigma\cdot\vec q}{|\vec q|} \otimes \frac{\vec\sigma\cdot\vec q}{|\vec q|}\right) \chi^1_M = \frac{\sqrt{4\pi}}{3}\, (Y^M_{011}(\Omega) + \sqrt{8}\, Y^M_{211}(\Omega)) \qquad (M=0,\pm 1) \tag{37}$$

to the positive parity BSA combining (14) and (28):

$$\Psi^{(JM)}(P,q) = -\frac{i(g^{(J)}(P))^2}{(2\pi)^4} S_{F2}(P,q) \sum_j v_j(q)\, g_j\, \Gamma_j\, X^{(JM)}_{+\,j}(P) \qquad (j=s,v,p) \tag{38}$$

leads to the following decomposition of the positive parity BSA in partial waves:

$$\Psi^{(00)}(M_d, \vec 0; q) = \begin{pmatrix} \Psi_{^1S_0^+}(M_d\,; q_0, |\vec q|)\, Y^0_{000}(\Omega) \\ \Psi_{^3P_0^+}(M_d\,; q_0, |\vec q|)\, Y^0_{110}(\Omega) \\ \Psi_{^3P_0^-}(M_d\,; q_0, |\vec q|)\, Y^0_{110}(\Omega) \\ \Psi_{^1S_0^-}(M_d\,; q_0, |\vec q|)\, Y^0_{000}(\Omega) \end{pmatrix}$$

$$\Psi^{(1M)}(M_d\,,\vec{0};q) \;=\; \begin{pmatrix} \Psi_{\,^3S_1^+}(M_d\,;q_0,|\vec{q}|)\,Y_{011}^M(\Omega) \;+\; \Psi_{\,^3D_1^+}(M_d\,;q_0,|\vec{q}|)\,Y_{211}^M(\Omega) \\ \Psi_{\,^3P_1^+}(M_d\,;q_0,|\vec{q}|)\,Y_{111}^M(\Omega) \;+\; \Psi_{\,^1P_1^+}(M_d\,;q_0,|\vec{q}|)\,Y_{101}^M(\Omega) \\ \Psi_{\,^3P_1^-}(M_d\,;q_0,|\vec{q}|)\,Y_{111}^M(\Omega) \;+\; \Psi_{\,^1P_1^-}(M_d\,;q_0,|\vec{q}|)\,Y_{101}^M(\Omega) \\ \Psi_{\,^3S_1^-}(M_d\,;q_0,|\vec{q}|)\,Y_{011}^M(\Omega) \;+\; \Psi_{\,^3D_1^-}(M_d\,;q_0,|\vec{q}|)\,Y_{211}^M(\Omega) \end{pmatrix} \tag{39}$$

in the rest frame of the deuteron.

3 The equal-time-wavefunction $\Phi\,(P,\vec{q})$ (ETW)

To correlate the determined BSAs to conventional bound state wavefunctions, the corresponding ETWs $\Phi\,(P,\vec{q})$ are computed by:

$$\Phi\,(P,\vec{q}) \;:=\; \int dq^0\;\Psi(P,q)\,e^{i\,q^0x^0}\Big|_{x^0\,=\,0} \;\stackrel{!}{=}\; \int dq^0\;\Psi(P,q) \tag{40}$$

With the definition of:

$$I_{(i)}(P,\vec{q}) \;:=\; \int dq^0\;S_{F2}\,(P,q)\,v_i(q) \qquad (i=s,v,p) \tag{41}$$

the BSA (38) changes in the rest frame to:

$$\Phi^{(JM)}(M_d\,,\vec{0};\vec{q}) = -\frac{i(g^{(J)}(M_d\,,\vec{0}))^2}{(2\pi)^4}\sum_j I_{(j)}(M_d\,,\vec{0};\vec{q})\,g_j\,\Gamma_j\,X_{+j}^{(JM)}(M_d\,,\vec{0}) \quad (j=s,v,p) \tag{42}$$

The decomposition in partial waves gives for the positive parity ETW in the rest frame:

$$\Phi^{(00)}(M_d\,,\vec{0};\vec{q}) \;=\; \begin{pmatrix} \Phi_{\,^1S_0^+}(M_d\,;|\vec{q}|)\,Y_{000}^0(\Omega) \\ \Phi_{\,^3P_0^+}(M_d\,;|\vec{q}|)\,Y_{110}^0(\Omega) \\ \Phi_{\,^3P_0^-}(M_d\,;|\vec{q}|)\,Y_{110}^0(\Omega) \\ \Phi_{\,^1S_0^-}(M_d\,;|\vec{q}|)\,Y_{000}^0(\Omega) \end{pmatrix} \tag{43}$$

$$\Phi^{(1M)}(M_d\,,\vec{0};\vec{q}) \;=\; \begin{pmatrix} \Phi_{\,^3S_1^+}(M_d\,;|\vec{q}|)\,Y_{011}^M(\Omega) & + & \Phi_{\,^3D_1^+}(M_d\,;|\vec{q}|)\,Y_{211}^M(\Omega) \\ \Phi_{\,^3P_1^+}(M_d\,;|\vec{q}|)\,Y_{111}^M(\Omega) & + & \Phi_{\,^1P_1^+}(M_d\,;|\vec{q}|)\,Y_{101}^M(\Omega) \\ \Phi_{\,^3P_1^-}(M_d\,;|\vec{q}|)\,Y_{111}^M(\Omega) & + & \Phi_{\,^1P_1^-}(M_d\,;|\vec{q}|)\,Y_{101}^M(\Omega) \\ \Phi_{\,^3S_1^-}(M_d\,;|\vec{q}|)\,Y_{011}^M(\Omega) & + & \Phi_{\,^3D_1^-}(M_d\,;|\vec{q}|)\,Y_{211}^M(\Omega) \end{pmatrix} \tag{44}$$

The free two–fermion–propagator can be expanded with the help of the two fermion energy projection operators $\Lambda^{\pm\oplus}(\vec{q}) := \Lambda^{\pm}(\vec{q}) \otimes \Lambda^{\oplus}(-\vec{q})$:

$$S_{F2}(M_d\,,\vec{0}\,;q) \quad =$$

$$= \left(\sum_{\pm\oplus} \frac{\Lambda^{\pm\oplus}(\vec{q})}{((E_N+q^0)\pm(-\omega(\vec{q})+i\varepsilon))\,((E_N-q^0)\oplus(-\omega(\vec{q})+i\varepsilon))}\right)\left(\gamma^0\otimes\gamma^0\right) \tag{45}$$

$$(E_N := \frac{M_d}{2} \quad , \quad \omega(\vec{q}) := \sqrt{|\vec{q}|^2+m^2} \quad , \quad \omega_i(\vec{q}) := \sqrt{|\vec{q}|^2+\Lambda_i^2} \quad)$$

Upon performing the integration in (41), we find in the rest frame of the bound state:

$$\begin{aligned} I_{(i)}(M_d\,,\vec{0}\,;\vec{q}) \;=\; \frac{2\pi}{i} \cdot \Big(\;& w_{(i)}^{++}(E_N;\vec{q})\,\Lambda^{++}(\vec{q}) \;+ \\ +\;& w_{(i)}^{+-}(E_N;\vec{q})\,\Lambda^{+-}(\vec{q}) \;+ \\ +\;& w_{(i)}^{-+}(E_N;\vec{q})\,\Lambda^{-+}(\vec{q}) \;+ \\ +\;& w_{(i)}^{--}(E_N;\vec{q})\,\Lambda^{--}(\vec{q})\Big)\left(\gamma^0\otimes\gamma^0\right) \end{aligned} \tag{46}$$

with

$$\begin{aligned} w_{(i)}^{++}(E_N;\vec{q}) \;:=\; & \frac{\Lambda_i}{2\,(E_N-\omega(\vec{q}))\,((E_N-\omega(\vec{q}))^2-\omega_i^2(\vec{q}))} \;+ \\ +\; & \frac{\Lambda_i}{((E_N-\omega(\vec{q}))^2-\omega_i^2(\vec{q}))} \cdot \frac{1}{2\,\omega_i(\vec{q})} \end{aligned}$$

$$w^{+-}_{(i)}(E_N;\vec{q}) \;:=\; \frac{\Lambda_i}{(E_N^2-(\omega(\vec{q})+\omega_i(\vec{q}))^2)}\cdot\frac{1}{2\,\omega_i(\vec{q})}$$

$$w^{-+}_{(i)}(E_N;\vec{q}) \;:=\; \frac{\Lambda_i}{(E_N^2-(\omega(\vec{q})+\omega_i(\vec{q}))^2)}\cdot\frac{1}{2\,\omega_i(\vec{q})}$$

$$w^{--}_{(i)}(E_N;\vec{q}) \;:=\; \frac{-\Lambda_i}{2\,(E_N+\omega(\vec{q}))\,((E_N+\omega(\vec{q}))^2-\omega_i^2(\vec{q}))} \quad +$$

$$+\quad \frac{\Lambda_i}{((E_N+\omega(\vec{q}))^2-\omega_i^2(\vec{q}))}\cdot\frac{1}{2\,\omega_i(\vec{q})} \tag{47}$$

Partial waves of the ETW in the rest frame (in arbitary units) are shown in fig. 1 (scalar interaction) and fig. 2 (full interaction) for one selected eigensolution of the BSE. Interaction parameters and binding energy are given in units of the nucleon mass, coupling constants are dimensionless.

4 Elastic e-d-scattering in the IA and deuteron form factors

Elastic electron–deuteron–scattering in impulse approximation (IA) is considered, as a first application following a similar route as in M.J. Zuilhof and J.A. Tjon 1980 [1]:

$$\begin{aligned} &< P_f, M_f | j_d^\mu | P_i, M_i > \quad = \\ &\overset{!}{=}\; 2\pi i\,\frac{e}{M_d}\int d^4q\;\tilde{\Psi}_{M_f}(P_f,\,q+\frac{1}{2}k)\;\Big(\Gamma^\mu(k^2)\;\otimes\;(\frac{1}{2}\;\not{P}_i-\not{q}-m)\Big)\;\Psi_{M_i}(P_i,q) \end{aligned} \tag{48}$$

Thereby, M_i and M_f are the polarization of the deuteron before and after scattering, respectively, k is the four–momentum transfer by the photon. The vertex function $\Gamma^\mu(k^2)$ is evaluated with the isoscalar (dipole) formfactors $F_1^S(k^2)$ and $F_2^S(k^2)$ of the nucleon:

$$\Gamma^\mu(k^2) \quad = \quad \gamma^\mu\;F_1^S(k^2)\;+\;\frac{i}{2m}\,\sigma^{\mu\nu}\,k_\nu\,F_2^S(k^2) \tag{49}$$

For the nucleonic formfactors $F_1^S(k^2)$ and $F_2^S(k^2)$ we use fits of Iachello et al. [4].
In the Breit–frame the charge, magnetic and quadrupole formfactors $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ are obtained from three independent matrix elements of the deuteron current, e.g. $(\eta := -k^2/(2\,M_d)^2)$:

$$e\,\sqrt{1+\eta}\,F_C \quad = \quad \frac{1}{3} < P_f, 0 | j_d^0 | P_i, 0 > \,+\, \frac{2}{3} < P_f, 1 | j_d^0 | P_i, 1 > \tag{50}$$

$$e\,\frac{k}{2\,M}\sqrt{\frac{1+\eta}{2}}\,F_M \quad = \quad < P_f, 1 | j_d^1 | P_i, 0 > \tag{51}$$

$$2\,\eta\,e\,\sqrt{1+\eta}\,F_Q \quad = \quad <P_f,0|j_d^0|P_i,0> \;-\; <P_f,1|j_d^0|P_i,1> \tag{52}$$

The formfactors $A(k^2)$, $B(k^2)$ of the *Rosenbluth formula* are calculated easily by:

$$A(k^2) = F_C^2 + \frac{8}{9}\,\eta^2\,F_Q^2 + \frac{2}{3}\,\eta\,F_M^2 \quad , \quad B(k^2) = \frac{4}{3}\,\eta\,(\eta+1)\,F_M^2 \tag{53}$$

with the appropriately boosted BSAs and deuteron currents from the deuteron rest frame to the Breit–frame.
As a characteristic result, formfactors are shown in fig. 3 and fig. 4 for the following set of parameters:

$$\Lambda_s \;=\; \Lambda_v \;= \Lambda_p \;=\; 0.24 \quad , \quad g_v = 1.02 \quad , \quad g_0 = 1. \quad , \quad g_p = 1.37 \quad , \quad E_B = 2.371 \cdot 10^{-3}$$

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Figure 1: Partial wave decomposition of the $J^\pi = 1^+$ Bethe–Salpeter amplitude for $\Lambda_s = \Lambda_v = \Lambda_p = 0.34$, $g_v = g_0 = g_p = 0$, $E_B = 0.002$

Figure 2: As Fig. 1, however for the parameters: $\Lambda_s = \Lambda_v = \Lambda_p = 0.34$, $g_v = 1.2$, $g_0 = 1.$, $g_p = -0.99$, $E_B = 0.002$

Figure 3: Formfactors $F_C(k_z^2), F_M(k_z^2), F_Q(k_z^2)$ of the deuteron for the parameters: $\Lambda_s = \Lambda_v = \Lambda_p = 0.24$, $g_v = 1.02$, $g_0 = 1.$, $g_p = 1.37$, $E_B = 2.371 \cdot 10^{-3}$

Figure 4: Formfactors $A(k_z^2), B(k_z^2)$ of the deuteron (parameters are chosen as in Fig. 3)