# Bethe–Salpeter–Approach to Relativistic Two–Fermion–Systems with Nonstatic Interaction

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The goal of our work is to study effects of relativistic both weakly and strongly bound systems.

Our test particle is the deuteron (mass $M$) as a weakly bound two–fermion–system of the two constituents (mass $m$, four–momentum $p_1$ and $p_2$) ($P := p_1 + p_2$ , $q := \frac{1}{2}(p_1 - p_2)$) with:

$$S_{F2}^{-1}(P,q)\,\Psi(P,q) = -\frac{i}{(2\pi)^4}\int d^4k\; K(P,q\,;k)\,\Psi(P,k)$$

and

$$S_{F2}(P,q) = \Big[ (\frac{1}{2}\not{P} + \not{q} - m + i\varepsilon) \otimes (\frac{1}{2}\not{P} - \not{q} - m + i\varepsilon) \Big]^{-1}$$

(to begin with we drop self–energy corrections to the nucleon mass).

The most general interaction kernel includes the 16 two body Dirac covariants $\Gamma_j$:

$$K(P\,,q\,;k) \;=\; g^2 \cdot \Big\{ \sum_j g_j \;\; f_j(P\,,q\,;k)\,\Gamma_j \Big\}$$

For the practical calculation we use as interaction, respecting covariance, retardation effects and integrability of the Bethe–Salpeter–Equation (BSE), the following separable kernel ($q^2 = (q^0)^2 - |\vec{q}|^2$):

$$K(q\,;k) \;=\; g^2 \cdot \Big\{ \sum_j g_j \frac{\Lambda_j}{q^2 - \Lambda_j^2 + i\varepsilon} \frac{\Lambda_j}{k^2 - \Lambda_j^2 + i\varepsilon}\,\Gamma_j \Big\} \qquad (j = s, v, ps)$$

(Different approaches to BSE with separable kernels can be found in the literature[1]).

With this kernel the BSE is transformed into an eigenvalue problem and then solved in the rest frame of the two particle system. Input parameters are the binding energy $E_B$, the interaction parameters $\Lambda_s$, $\Lambda_v$, $\Lambda_{ps}$ and the coupling constants $g_s$, $g_v$, $g_{ps}$. The calculated eigenvalues are the coupling constants $g^2$ (for the different eigensolutions of the deuteron wavefunction), the eigenvectors are closely related to the corresponding Bethe–Salpeter–Amplitudes $\Psi(P,q)$. Only the positive parity eigensolutions of the Bethe–Salpeter–Amplitutes (BSAs) are considered. To correlate the determined BSAs to ordinary quantummechanical deuteron wavefunctions, the corresponding *equal–time–wavefunctions* $\Phi\,(P,\vec{q})$ are computed by:

$$\Phi\,(P,\vec{q}) \quad := \quad \int dq^0\;\; \Psi(P,q)\, e^{i\,q^0 x^0}\Big|_{x^0\,=\,0} \quad \stackrel{!}{=} \quad \int dq^0\;\; \Psi(P,q)$$

A decomposition of BSAs and equal–time–wavefunctions in partial waves of different angular momentum and parity quantum numbers has been performed.

Elastic electron–deuteron–scattering in impulse approximation (IA) is considered, as a first application following a similar route as in M.J. Zuilhof and J.A. Tjon 1980 [2] with:

$$
\begin{aligned}
< P_f, M_f | j_d^\mu | P_i, M_i > \quad &= \\
&\stackrel{!}{=} \; 2\pi i\, e\; M^{-1} \int d^4 q\; \tilde{\Psi}_{M_f}(P_f,\, q + \frac{1}{2}\, k) \; \Big( \Gamma^\mu(k^2) \,\otimes\, ( \frac{1}{2}\; \not{P}_i - \not{q} - m ) \Big) \; \Psi_{M_i}(P_i, q)
\end{aligned}
$$

Thereby, $M_i$ is the polarization of the deuteron before, $M_f$ after the scattering, $k$ is the four–momentum transfer by the photon. The vertex function $\Gamma^\mu(k^2)$ is evaluated with the isoscalar (dipole) formfactors $F_1^S(k^2)$ and $F_2^S(k^2)$ of the nucleon:

$$
\Gamma^\mu(k^2) \quad = \quad \gamma^\mu\, F_1^S(k^2) \,+\, \frac{i}{2m}\, \sigma^{\mu\nu}\, k_\nu\, F_2^S(k^2)
$$

In the Breit–frame of elastic electron–deuteron–scattering the charge, magnetic and quadrupole formfactors $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ are obtained from three independent matrix elements of the deuteron current, e.g. $(\eta := -k^2/(2\, M)^2)$:

$$
\begin{aligned}
e\, \sqrt{1+\eta}\; F_C \quad &= \quad \frac{1}{3} < P_f, 0 | j_d^0 | P_i, 0 > \,+\, \frac{2}{3} < P_f, 1 | j_d^0 | P_i, 1 > \\
e\, \frac{k}{2\, M} \sqrt{\frac{1+\eta}{2}}\; F_M \quad &= \quad < P_f, 1 | j_d^1 | P_i, 0 > \\
2\, \eta\, e\, \sqrt{1+\eta}\; F_Q \quad &= \quad < P_f, 0 | j_d^0 | P_i, 0 > \,-\, < P_f, 1 | j_d^0 | P_i, 1 >
\end{aligned}
$$

The formfactors $A(k^2)$, $B(k^2)$ of the *Rosenbluth formula* are calculated easily by:

$$
A(k^2) = F_C^2 + \frac{8}{9}\, \eta^2\, F_Q^2 + \frac{2}{3}\, \eta\, F_M^2 \quad , \quad B(k^2) = \frac{4}{3}\, \eta\, (\eta + 1)\; F_M^2
$$

with the appropriately boosted BSAs and deuteron currents from the deuteron rest frame to the Breit–frame.

Extensions of this formalism to mesons as $q\bar{q}$ or baryons as quark–diquark $q\,(2q)_{\mathrm{Bose}}$ systems are under way.

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[1] G. Rupp, Nucl. Phys. **A 509** (1990) p. 131c, see also FEW BODY XII
G. Rupp, J.A. Tjon, Phys. Rev. **C 41**, No. 2 (1990) p. 472

[2] M.J. Zuilhof, J.A. Tjon, Phys. Rev. **C 22**, No. 6 (1980) p. 2369