On (non-Hermitian) Lagrangeans in (particle) physics and their dynamical generation *)

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On the basis of a new method for the derivation of the effective action the nonperturbative concept of "dynamical generation" is explained. A non-trivial, non-Hermitian and PT-symmetric solution for Wightman's scalar field theory in four dimensions is dynamically generated, rehabilitating Symanzik's precarious ϕ^4-theory with a negative quartic coupling constant as a candidate for an asymptotically free theory of strong interactions. Finally it is shown making use of the dynamical generation that a Symanzik-like field theory with scalar confinement for the theory of strong interactions can be even suggested by experiment.

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1 Dynamical generation of Lagrangeans

1.1 The concept of dynamical generation

The concept and terminology of "dynamical generation" occurred to us for the first time explicitly in the context of the (one-loop) "dynamical generation" of the Quark-Level Linear Sigma Model by M.D. Scadron and R. Delbourgo [1].

A particularly important issue in the process of quantizing a theory given by some classical Lagrangean is the aspect of renormalization and renormalizability [2]. The process of renormalization is typically performed — after choosing some valid regularization scheme (See e.g. Ref. [3]) — by adding to the classical Lagrangean divergent counterterms, which remove divergencies that would otherwise show up in the unrenormalized effective action. Naively one might think that renormalization only affects terms belonging to the same order of perturbation theory in some coupling constant, while other parameters of the same Lagrangean do not interfere. The underlying philosophy would here be that in a quantum theory distinct parameters (e.g. masses, couplings) in a Lagrangean can be considered — like in a classical Lagrangean — to a great extent uncorrelated, as long as the Lagrangean is renormalizable. It appears that this philosophy seems to work quite well, when it has to renormalize logarithmic divergencies. That the situation is not so easy can be seen from the formalism needed to renormalize non-Abelian vector fields [4]. In

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such theories the values of the coupling constants responsible for the self-interaction of three-vector fields and of four-vector fields are highly correlated due to the need to cancel appearing quadratic divergencies in the process of summing up diagrams of different loop order (in particular to achieve here the fundamental principle of gauge invariance). If this were not like that, their values could be independently chosen and therefore also independently renormalized. We see here a first example of "dynamical" generation or interrelation of two otherwise independent parameters in a Lagrangean due to the requirement of renormalizability, which affects here also the *cancellation of quadratic divergencies*. Furthermore we learn that "dynamical generation" typically interrelates *seemingly uncorrelated parameters of the Lagrangean* and *different loop orders* [1]). Renormalizable theories with scalar fields only seem naively to have the priviledge, not to be affected by the problem faced by non-Abelian gauge theories, as the quadratic divergencies seem to be removable before entering the renormalization of logarithmic divergencies. Hence it naively seems that — as long as a Lagrangean with scalar fields only is in a classical sense considered to be renormalizable — different parameters of the Lagrangean can be renormalized individually (up to constraints resulting from multiplicative renormalization). It is exactly this misbelief, which leads indeed to the triviality of scalar field theories like the textbook ϕ^4 theory or even to intimately related Abelian gauge theories like QED, if not "dynamically generated". If instead the respective theories are "dynamically generated" one *does* find — besides the trivial solution — also non-trivial choices of the their parameter space, which survive the renormalization process without running into triviality. Interestingly in many cases such non-trivial solutions are found in the sector of the parameter space related to a *PT-symmetric* [6], yet not necessarily to a *Hermitian* non-trivial theory [2]). In order to "dynamically generate" a theory (e.g. like the supersymmetric Wess-Zumino model [14]) on the basis of some tentative classical Lagrangean, we have to

[1]) Most probably the most outstanding example for dynamically generated theories are theories containing supersymmetry. This is reflected by the fact that supersymmetric theories typically contain a minimum of parameters, quadratic divergencies cancel exactly without extra renormalization (see e.g. Ref. [5]), and the renormalization of logarithmic divergencies at one-loop order yields simultaneously an automatic renormalization of all higher-loop orders. That observation led already to (non-conclusive) speculations about the question, whether all theories cancelling quadratic divergencies must be supersymmetric (see e.g. Refs. [5]). In certain situations some — not necessarily supersymmetric — theories may display even strong cancellations on the level of logarithmic divergencies. In such "bootstrapping" theories physics is determined already at "tree-level", as cancelling loop-contributions show up to be marginal.

[2]) Before proceeding we want to deliver here also some warning about some common regularization schemes used which must not to be used in the context of "dynamical generation": Most important information about divergencies underlying a theory is contained in tadpole diagrams; hence any kind of artificial normal ordering or suppression of important surface terms will erase information needed to dynamically generate the theory and will lead therefore to wrong conclusions (See e.g. the discussion in Refs. [7, 8]). As dimensional regularization erases or changes several important divergent diagrams like the massless tadpole (see e.g. Ref. [9]) or the quadratic divergence in the sunset/sunrise graph (see e.g. the dimensional regularization calculations performed in Refs. [10–12], or on p. 114 ff in Ref. [13]), it should not be used to dynamically generate a theory! According to our experience cutoff regularization — if correctly used — seems to yield always correct and most compact results compared to other regularization schemes.

perform two steps: first we have to construct the terms in the effective action which are causing non-logarithmic divergencies (i.e. linear, quadratic, and higher-order divergencies) in all connected Feynman diagrams, which can be constructed from the theory; then we have to relate and choose the parameters entering these terms of the effective action such, that all non-logarithmic divergencies cancel.[3])

1.2 New method for the derivation of the effective action and its Lagrangean

A powerful method to construct the effective action has been known at least since the benchmarking work of S. Coleman and E. Weinberg [15] and R. Jackiw [16]. Unfortunately it is for our purposes not very convenient, as the determination of desired terms of the effective action responsible for leading singularities typically requires the simultaneous tedious evaluation of many other terms, which do not alter the discussion. This is why we want to propose here a different — to our best knowledge — new and more pragmatic approach yielding equivalent results compared to the formalism of S. Coleman, E. Weinberg, and R. Jackiw. Without loss of generality we want to explain our simple method here on the basis of an example, the generalization of which is quite straight forward.

Let us start with the interaction part $\mathcal{S}_{\text{int}} = \int \mathrm{d}^4 z\; \mathcal{L}_{\text{int}}(\vec{\phi}(z), \partial_z \vec{\phi}(z))$ of an action $\mathcal{S}$ of N interacting Klein–Gordon fields $\phi_1(z)$, ..., $\phi_N(z)$. Then the interaction part of the effective action responsible for a process involving n external legs is calculated by the connected ($\langle \ldots \rangle_{\mathrm{c}}$) time-ordered vacuum expectation value of the Dyson operator, where contractions are to be performed over all fields except n fields ("except ϕ^n"), which remain to be contracted with creation or annihilation operators appearing in initial or final states, i.e.:

$$\begin{aligned}
\frac{\mathrm{i}}{1!}\,\mathcal{S}_{\text{eff}} &= \langle 0|\, T[\,\exp(\mathrm{i}\,\mathcal{S}_{\text{int}})]\,|0\rangle_{\mathrm{c}}\,|_{\text{except }\phi^n} \\
&= \frac{\mathrm{i}}{1!}\;\langle 0|\,T[\,\mathcal{S}_{\text{int}}]\,|0\rangle_{\mathrm{c}}\,|_{\text{except }\phi^n} + \frac{\mathrm{i}^2}{2!}\;\langle 0|\,T[\,\mathcal{S}_{\text{int}}\,\mathcal{S}_{\text{int}}]\,|0\rangle_{\mathrm{c}}\,|_{\text{except }\phi^n} \\
&\quad + \frac{\mathrm{i}^3}{3!}\;\langle 0|\,T[\,\mathcal{S}_{\text{int}}\,\mathcal{S}_{\text{int}}\,\mathcal{S}_{\text{int}}]\,|0\rangle_{\mathrm{c}}\,|_{\text{except }\phi^n} + \ldots\,. \qquad (1)
\end{aligned}$$

The method is proved by making heavy use of the following identity (inserted between initial and final states $|i\rangle$ and $\langle f|$, respectively) found e.g. on p. 44 in a well-known book by C. Nash [17], i.e.:

$$\begin{aligned}
&\langle f|\,T[\,\exp(\mathrm{i}\,\mathcal{S}_{\text{int}})]\,|i\rangle \;=\; \langle f|\exp\left(\left[\frac{1}{2}\,\langle\phi^2\rangle\,\frac{\delta^2}{\delta\phi^2}\right]\right)\,:\exp(\mathrm{i}\,\mathcal{S}_{\text{int}}):\,|i\rangle \\
&= \langle f|\exp\left(\left[\frac{1}{2}\,\langle\phi^2\rangle\,\frac{\delta^2}{\delta\phi^2}\right]\right)\left(:\frac{\mathrm{i}}{1!}\,\mathcal{S}_{\text{int}}: + :\frac{\mathrm{i}^2}{2!}\,\mathcal{S}_{\text{int}}\mathcal{S}_{\text{int}}: + \ldots\right)|i\rangle \\
&= \langle f|\left[\frac{\mathrm{i}}{1!}\left\{:\mathcal{S}_{\text{int}}: + \frac{1}{1!}\left[\frac{1}{2}\,\langle\phi^2\rangle\,\frac{\delta^2}{\delta\phi^2}\right]:\mathcal{S}_{\text{int}}: + \frac{1}{2!}\left[\frac{1}{2}\,\langle\phi^2\rangle\,\frac{\delta^2}{\delta\phi^2}\right]^2 :\mathcal{S}_{\text{int}}: + \ldots\right\} + \right.
\end{aligned}$$

[3]) One feels the need to remark that the very existence of a dynamically generated theory is not always guaranteed, as the procedure of dynamical generation is intimately related to renormalization and — even more — is strongly constraining the parameters of the effective action.

$$+\frac{\mathrm{i}^2}{2!}\Big\{:\mathcal{S}^2_{\rm int}:+\frac{1}{1!}\left[\frac{1}{2}\left\langle\phi^2\right\rangle\frac{\delta^2}{\delta\phi^2}\right]:\mathcal{S}^2_{\rm int}:+\frac{1}{2!}\left[\frac{1}{2}\left\langle\phi^2\right\rangle\frac{\delta^2}{\delta\phi^2}\right]^2:\mathcal{S}^2_{\rm int}:+\ldots\Big\}$$
$$+\frac{\mathrm{i}^3}{3!}\Big\{:\mathcal{S}^3_{\rm int}:+\frac{1}{1!}\left[\frac{1}{2}\left\langle\phi^2\right\rangle\frac{\delta^2}{\delta\phi^2}\right]:\mathcal{S}^3_{\rm int}:+\frac{1}{2!}\left[\frac{1}{2}\left\langle\phi^2\right\rangle\frac{\delta^2}{\delta\phi^2}\right]^2:\mathcal{S}^3_{\rm int}:+\ldots\Big\}$$
$$+\ \ldots\Big]\,|i\rangle\ , \qquad (2)$$

where we have defined for convenience the short-hand notation

$$\left[\frac{1}{2}\left\langle\phi^2\right\rangle\frac{\delta^2}{\delta\phi^2}\right]\equiv\frac{1}{2}\sum_{i_1,i_2=1}^{N}\int \mathrm{d}^4z_1\,\mathrm{d}^4z_2\,\langle 0|\,T[\,\phi_{i_1}(z_1)\,\phi_{i_2}(z_2)]\,|0\rangle\,\frac{\delta^2}{\delta\phi_{i_2}(z_2)\delta\phi_{i_1}(z_1)}\ . \qquad (3)$$

The identity (see e.g. p. 49 in Ref. [17]) and method is easily extended to fermions, i.e. Grassmann fields $\psi_1(z)$, …, $\psi_N(z)$, by replacing $\left[\frac{1}{2}\left\langle\phi^2\right\rangle\delta^2/\delta\phi^2\right]$ by

$$\left[\left\langle\psi\,\bar\psi\right\rangle\frac{\delta^2}{\delta\bar\psi\delta\psi}\right]\equiv\sum_{i_1,i_2=1}^{N}\int \mathrm{d}^4z_1\,\mathrm{d}^4z_2\,\langle 0|\,T[\psi_{i_1}(z_1)\bar\psi_{i_2}(z_2)]\,|0\rangle\,\frac{\delta^2}{\delta\bar\psi_{i_2}(z_2)\delta\psi_{i_1}(z_1)}\ . \qquad (4)$$

Convince yourself, that the method reproduces Coleman's and Weinberg's loop-expansion [15] for a simple massless ϕ^4-theory with $\mathcal{S}_{\rm int}=\int \mathrm{d}^4z\,(-\lambda/4!)\,\phi^4(z)$. [4])

2 Applications

2.1 A.S. Wightman's (non-)trivial and K. Symanzik's precarious ϕ^4 theory

In this section we want to shortly sketch the steps to dynamically generate the "Scalar Wightman Theory in 4 Space-Time Dimensions" [19] (see also Ref. [13]).

[4]) We show here only the most important steps of the derivation:

$$\begin{aligned}\frac{\mathrm{i}}{1!}\,\mathcal{S}_{\rm eff} &= \sum_{n=1}^{\infty}\frac{\mathrm{i}^n}{n!}\,\langle 0|\,T[\,\mathcal{S}^n_{\rm int}]\,|0\rangle_{\rm c}\,|_{{\rm except}\ \phi^{2n}}=\sum_{n=1}^{\infty}\frac{\mathrm{i}^n}{n!}\,\frac{1}{n!}\left[\frac{1}{2}\left\langle\phi^2\right\rangle\frac{\delta^2}{\delta\phi^2}\right]^n\mathcal{S}^n_{\rm int}\\
&=\sum_{n=1}^{\infty}\frac{\mathrm{i}^n}{n!}\,\frac{1}{n!}\int \mathrm{d}^4z_1\ldots \mathrm{d}^4z_n\,\frac{n!(n-1)!}{2}\,\left(-\frac{\lambda}{2!}\right)^n\phi^2(z_1)\ldots\phi^2(z_n)\\
&\quad\times\langle 0|\,T[\,\phi(z_1)\,\phi(z_2)]\,|0\rangle\,\langle 0|\,T[\,\phi(z_2)\,\phi(z_3)]\,|0\rangle\ldots\langle 0|\,T[\,\phi(z_n)\,\phi(z_1)]\,|0\rangle\\
&=\sum_{n=1}^{\infty}\left(\frac{\lambda}{2!}\right)^n\frac{1}{2n}\int \mathrm{d}^4z_1\ldots \mathrm{d}^4z_n\,\phi^2(z_1)\ldots\phi^2(z_n)\\
&\quad\times\int\frac{\mathrm{d}^4p_{12}}{(2\pi)^4}\frac{\mathrm{d}^4p_{23}}{(2\pi)^4}\ldots\frac{\mathrm{d}^4p_{n1}}{(2\pi)^4}\,\frac{\mathrm{e}^{-\mathrm{i}p_{12}(z_1-z_2)}\mathrm{e}^{-\mathrm{i}p_{23}(z_2-z_3)}\ldots\mathrm{e}^{-\mathrm{i}p_{n1}(z_n-z_1)}}{(p^2_{12}+\mathrm{i}\varepsilon)(p^2_{23}+\mathrm{i}\varepsilon)\ldots(p^2_{n1}+\mathrm{i}\varepsilon)}\\
&=\int \mathrm{d}^4z\left(\sum_{n=1}^{\infty}\left(\frac{\lambda}{2!}\right)^n\frac{1}{2n}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\left(\frac{\phi^2(0)}{p^2+\mathrm{i}\varepsilon}\right)^n+\text{non-local terms}\right).\end{aligned} \qquad (5)$$

Some of the resulting non-local terms are nicely discussed e.g. in Ref. [18].

As we shall see below, the dynamical generation of this so-called ϕ^4 theory yields — besides the well-known "trivial" solution — the "precarious" [20] non-trivial solution suggested by K. Symanzik [21] being non-Hermitian and — under certain circumstances — also PT-symmetric [6].

To dynamically generate a ϕ^N-theory upto $N = 4$, we start from the following lowest-order action containing just a three-point interaction:

$$\begin{aligned}\mathcal{S}_{(0)} &= \int \mathrm{d}^4 z \,\left\{ \frac{1}{2}\Big((\partial\,\phi_{(0)}(z))^2 - m^2_{(0)}\;\phi^2_{(0)}(z)\Big) \;-\; \frac{1}{3!}\, g_{(0)}\;\phi^3_{(0)}(z)\right\} \\ &= \mathcal{S}_{(0)}[\,(\partial\,\phi)^2\,] + \mathcal{S}_{(0)}[\,\phi^2\,] + \mathcal{S}_{(0)}[\,\phi^3\,]\,. \end{aligned} \tag{6}$$

In the first step we want to absorb by dynamical generation the finite one-loop correction to the ϕ^3-coupling into a renormalization of the three-point coupling, i.e.:

$$\begin{aligned}\frac{\mathrm{i}}{1!}\,\mathcal{S}_{(1)}[\,\phi^3\,] \;=\; &\frac{\mathrm{i}}{1!}\;\langle 0|T\Big[\mathcal{S}_{(0)}[\,\phi^3\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\text{except }\phi^3_{(0)}} \\ &+\frac{\mathrm{i}^3}{3!}\;\langle 0|T\Big[\mathcal{S}_{(0)}[\,\phi^3\,]\;\mathcal{S}_{(0)}[\,\phi^3\,]\;\mathcal{S}_{(0)}[\,\phi^3\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\text{except }\phi^3_{(0)}}\,. \end{aligned} \tag{7}$$

The next step is to dynamically generate on the basis of $\mathcal{S}_{(1)}[\,\phi^3\,]$ the term of the effective action quadratic in the fields $\phi_{(0)}(z)$ *assuming* the absence of quadratically divergent terms. [5]) The result of the previous steps is simple multiplicative coupling, wave function and mass renormalization, as we obtain as a whole (the omissions ("...") denote here non-local terms not relevant to our present discussion):

$$\begin{aligned}&\mathcal{S}_{(1)}[\,(\partial\phi)^2\,] + \mathcal{S}_{(1)}[\,\phi^2\,] + \mathcal{S}_{(1)}[\,\phi^3\,] \;= \\ &= \int \mathrm{d}^4 z\,\left(\frac{1}{2}\Big((\partial\,\phi_{(1)}(z))^2 - m^2_{(1)}\;\phi^2_{(1)}(z)\Big) - \frac{1}{3!}\,g_{(1)}\;\phi^3_{(1)}(z)\right) + \dots\,, \end{aligned} \tag{9}$$

with

$$\begin{aligned} g_{(1)} &= \bar{g}_{(0)}\Bigg/\left(1 - \frac{1}{32\,\pi^2}\,\frac{\bar{g}^2_{(0)}}{m^2_{(0)}}\right)^{3/2}\,,\quad \bar{g}_{(0)} = g_{(0)}\left(1 + \frac{1}{32\,\pi^2}\,\frac{g^2_{(0)}}{m^2_{(0)}}\right)\,, \\ \phi^2_{(1)}(z) &= \phi^2_{(0)}(z)\left(1 - \frac{1}{32\,\pi^2}\,\frac{\bar{g}^2_{(0)}}{m^2_{(0)}}\right)\,, \\ m^2_{(1)} &= m^2_{(0)}\left(1 + \frac{\mathrm{i}}{2}\frac{\bar{g}^2_{(0)}}{m^2_{(0)}}\int\frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{(p^2 - m^2_{(0)})^2}\right)\Bigg/\left(1 - \frac{1}{32\,\pi^2}\frac{\bar{g}^2_{(0)}}{m^2_{(0)}}\right)\,. \end{aligned} \tag{10}$$

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[5]) That is, we consider:

$$\begin{aligned}&\frac{\mathrm{i}}{1!}\,\Big(\mathcal{S}_{(1)}[\,(\partial\phi)^2\,] + \mathcal{S}_{(1)}[\,\phi^2\,]\Big) = \frac{\mathrm{i}}{1!}\;\langle 0|T\Big[\mathcal{S}_{(0)}[\,(\partial\phi)^2\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\text{except }\phi^2_{(0)}} \\ &+\frac{\mathrm{i}}{1!}\;\langle 0|T\Big[\mathcal{S}_{(0)}[\,\phi^2\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\text{except }\phi^2_{(0)}} + \frac{\mathrm{i}^2}{2!}\;\langle 0|T\Big[\mathcal{S}_{(1)}[\,\phi^3\,]\,\mathcal{S}_{(1)}[\,\phi^3\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\text{except }\phi^2_{(0)}}\,. \end{aligned} \tag{8}$$

If we renormalize this result through a suitable mass counter term yielding a log.-divergent gap-equation promoted e.g. by M.D. Scadron [22], i.e., by applying

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, \frac{1}{(p^2 - m^2_{(0)})^2} \;\longrightarrow\; + \frac{\mathrm{i}}{16\,\pi^2}\;, \tag{11}$$

then we have a bootstrapping situation for the mass, as there holds then $m^2_{(1)} = m^2_{(0)}$. Recall that the result has been obtained by *assuming* the absence, i.e. the *cancellation* of quadratically divergent terms in $\mathcal{S}_{(1)}[\,(\partial\phi)^2\,] + \mathcal{S}_{(1)}[\,\phi^2\,]$. In order to show now the absence of quadratically divergent terms for self-consistency reasons, we have first to dynamically generate on the basis of $g_{(1)}$ and $m_{(1)}$ the effective action for a four-point interaction of the field $\phi_{(0)}(z)$, and then test the cancellations of quadratic divergencies on the level of tadpoles and selfenergies. The effective action for a four-point interaction of the field $\phi_{(0)}(z)$ (expressed in terms of $\phi_{(1)}(z)$) is here dynamically generated *for simplicity just* up to order $g^4_{(1)}$ *assuming* again the absence of quadratically divergent terms, i.e.:

$$\begin{aligned}
&\frac{\mathrm{i}}{1!}\,\mathcal{S}_{(1)}[\,\phi^4\,] = \frac{\mathrm{i}}{1!}\left(\mathcal{S}^{\mathrm{tree}}_{(1)}[\,\phi^4\,] + \mathcal{S}^{\mathrm{loop}}_{(1)}[\,\phi^4\,]\right) = \\
&= \frac{\mathrm{i}^2}{2!}\,\langle 0|T\Big[\mathcal{S}_{(1)}[\,\phi^3\,]\,\mathcal{S}_{(1)}[\,\phi^3\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\mathrm{except}\;\phi^4_{(1)}} \\
&\quad + \frac{\mathrm{i}^4}{4!}\,\langle 0|T\Big[\mathcal{S}_{(1)}[\,\phi^3\,]\,\mathcal{S}_{(1)}[\,\phi^3\,]\,\mathcal{S}_{(1)}[\,\phi^3\,]\,\mathcal{S}_{(1)}[\,\phi^3\,]\Big]\,|0\rangle_{\mathrm{c}}\Big|_{\mathrm{except}\;\phi^4_{(1)}} \\
&= \frac{\mathrm{i}^2}{2!}\int \mathrm{d}^4 z_1\,\mathrm{d}^4 z_2 \left(-\frac{1}{3!}g_{(1)}\right)^2 3^2 \phi^2_{(1)}(z_1)\,\phi^2_{(1)}(z_2)\,\mathrm{i}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\frac{\mathrm{e}^{-\mathrm{i}p(z_1-z_2)}}{(p^2 - m^2_{(1)})} \\
&\quad + \frac{\mathrm{i}^4}{4!}\int \mathrm{d}^4 z_1 \mathrm{d}^4 z_2 \mathrm{d}^4 z_3 \mathrm{d}^4 z_4 \left(-\frac{1}{3!}g_{(1)}\right)^4 3(3!)^4 \phi_{(1)}(z_1)\phi_{(1)}(z_2)\phi_{(1)}(z_3)\phi_{(1)}(z_4)\,\mathrm{i}^4 \\
&\quad \times\int \frac{\mathrm{d}^4 p_{12}}{(2\pi)^4}\frac{\mathrm{d}^4 p_{23}}{(2\pi)^4}\frac{\mathrm{d}^4 p_{34}}{(2\pi)^4}\frac{\mathrm{d}^4 p_{41}}{(2\pi)^4}\,\frac{\mathrm{e}^{-\mathrm{i}p_{12}(z_1-z_2)}\mathrm{e}^{-\mathrm{i}p_{23}(z_2-z_3)}\mathrm{e}^{-\mathrm{i}p_{34}(z_3-z_4)}\mathrm{e}^{-\mathrm{i}p_{41}(z_4-z_1)}}{(p^2_{12} - m^2_{(1)})(p^2_{23} - m^2_{(1)})(p^2_{34} - m^2_{(1)})(p^2_{41} - m^2_{(1)})} \\
&= \frac{\mathrm{i}}{1!}\int \mathrm{d}^4 z_1\,\mathrm{d}^4 z_2 \left(-\frac{1}{4!}\right) 3\,g^2_{(1)}\;\phi^2_{(1)}(z_1)\,\phi^2_{(1)}(z_2)\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\;\frac{\mathrm{e}^{-\mathrm{i}p(z_1-z_2)}}{(p^2 - m^2_{(1)})} \\
&\quad + \frac{\mathrm{i}}{1!}\int \mathrm{d}^4 z\,\left(-\frac{1}{4!}\right) 3\,\mathrm{i}\,g^4_{(1)}\;\phi^4_{(1)}(z)\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\;\frac{1}{(p^2 - m^2_{(1)})^4} + \ldots \\
&= \frac{\mathrm{i}}{1!}\int \mathrm{d}^4 z \left(-\frac{1}{4!}\right)\left((-\,3)\frac{g^2_{(1)}}{m^2_{(1)}}\,\phi^4_{(1)}(z) + \left(-\frac{1}{32\,\pi^2}\right)\,\frac{g^4_{(1)}}{m^4_{(1)}}\,\phi^4_{(1)}(z)\right) + \ldots \\
&= \frac{\mathrm{i}}{1!}\int \mathrm{d}^4 z \left(\left(-\frac{1}{4!}\right)(-\,3)\frac{g^2_{(1)}}{m^2_{(1)}}\,\phi^4_{(1)}(z) + \left(-\frac{1}{4!}\,\lambda_{(1)}\right)\,\phi^4_{(1)}(z)\right) + \ldots\;.
\end{aligned} \tag{12}$$

As a result of this consideration we have

$$\mathcal{S}_{(1)} = \int d^4z \left(\frac{1}{2} \left((\partial \phi_{(1)}(z))^2 - m^2_{(1)} \phi^2_{(1)}(z) \right) - \frac{1}{3!} g_{(1)} \phi^3_{(1)}(z) - \frac{1}{4!} \lambda_{(1)} \phi^4_{(1)}(z) \right) + \dots , \tag{13}$$

with $\lambda_{(1)} = -g^4_{(1)}/(32\pi^2 m^4_{(1)})$ and the replacements made in Eq. (11). Let us see now, on the basis of this action, in how far quadratic divergencies cancel, as assumed in our approach from the beginning. Therefore we dynamically generate — for convenience — e.g. the effective action describing the sum of quadratically divergent tadpoles:

$$\begin{aligned}
\frac{\mathrm{i}}{1!} \mathcal{S}_{(1)}[\phi] &= \frac{\mathrm{i}}{1!} \langle 0|T\left[\mathcal{S}_{(1)}[\phi^3] \right] |0\rangle_c \Big|_{\text{except } \phi_{(1)}} \\
&\quad + \frac{\mathrm{i}^2}{2!} 2! \langle 0|T\left[\mathcal{S}^{\text{loop}}_{(1)}[\phi^4] \, \mathcal{S}_{(1)}[\phi^3] \right] |0\rangle_c \Big|_{\text{except } \phi_{(1)}} \\
&= \frac{\mathrm{i}}{1!} \int d^4z \left(-\frac{1}{3!} g_{(1)} \right) 3 \; \phi_{(1)}(z) \, \mathrm{i} \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2_{(1)})} \\
&\quad + \frac{\mathrm{i}}{1!} \int d^4z \left(-\frac{1}{3!} g_{(1)} \right) (-1) \, \lambda_{(1)} \; \phi_{(1)}(z) \\
&\quad \times \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \frac{d^4p_3}{(2\pi)^4} \frac{(2\pi)^4 \, \delta^4(p_1 + p_2 + p_3)}{(p^2_1 - m^2_{(1)})(p^2_2 - m^2_{(1)})(p^2_3 - m^2_{(1)})} .
\end{aligned} \tag{14}$$

To proceed further we extract shortly in the footnote the leading singularity structure of the occurring massive sunset/sunrise diagram, being particularly complicated due to the overlap of one quadratic divergence with three logarithmic divergences (see e.g. p. 78 ff in Ref. [17]). [6]) The expression for the leading divergence of

[6]) The safest and most compact discussion of the sunset/sunrise diagram is achieved in cutoff regularization, even though the full diagram in cutoff regularization has — to our present knowledge — never been calculated in a closed form. For a discussion of the finite part of the sunset/sunrise integral for non-zero external four-momentum on the basis of implicit renormalization see e.g. Ref. [3]. The leading divergent parts of the sunset/sunrise diagram for zero external four-momentum and equal masses have been determined in cutoff regularization in Ref. [23] to be

$$\begin{aligned}
&\int^\Lambda \frac{d^4p_1}{(2\pi)^4} \int^\Lambda \frac{d^4p_2}{(2\pi)^4} \int^\Lambda \frac{d^4p_3}{(2\pi)^4} \frac{(2\pi)^4 \, \delta^4(p_1 + p_2 + p_3)}{(p^2_1 - m^2)(p^2_2 - m^2)(p^2_3 - m^2)} = \\
&= -\left(\frac{1}{16\,\pi^2} \right)^2 \left(2\Lambda^2 + \frac{3}{2} m^2 \ln^2 \left(\frac{\Lambda^2}{m^2} \right) - 3m^2 \ln \left(\frac{\Lambda^2}{m^2} \right) + C\, m^2 \right) + O(\Lambda^{-2}) ,
\end{aligned} \tag{15}$$

while the integration constant C was numerically estimated in [23] to be approximately $C \approx 4$. After recalling $\int^\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)^2} = \frac{\mathrm{i}}{16\,\pi^2} \left(\ln \frac{\Lambda^2 + m^2}{m^2} - \frac{\Lambda^2}{\Lambda^2 + m^2} \right)$ and $\int^\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)} = \frac{-\mathrm{i}}{16\pi^2} \, m^2 \left(\frac{\Lambda^2}{m^2} - \ln \frac{\Lambda^2 + m^2}{m^2} \right)$, Eq. (15) is replaced for $\Lambda \to \infty$ and in the local limit by

$$I_{\text{sunset/sunrise}} = \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \frac{d^4p_3}{(2\pi)^4} \frac{(2\pi)^4 \, \delta^4(p_1 + p_2 + p_3)}{(p^2_1 - m^2)(p^2_2 - m^2)(p^2_3 - m^2)} =$$

the sunset/sunrise graph is then to be inserted into Eq. (14), yielding the following result for the local limit of the effective action describing tadpoles:

$$\mathcal{S}_{(1)}[\phi] = \int \mathrm{d}^4 z \left(-\frac{1}{3!}\, g_{(1)}\right) 3\,\mathrm{i}\,\phi_{(1)}(z) \times \left\{ \left(1 + \frac{2}{3}\frac{1}{16\pi^2}\lambda_{(1)}\right) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{(p^2 - m^2_{(1)})} - \mathrm{i}\left(\frac{1}{16\pi^2}\right)^2 m^2_{(1)} \frac{(4+C)}{3}\lambda_{(1)} \right\} + \ldots \,. \tag{18}$$

Simple inspection of this expression shows that the quadratic divergencies cancel on one hand for the well-known "trivial" solution $g_{(1)} = 0$. On the other hand the dynamically generated theory displays a non-trivial, precarious solution in the spirit of K. Symanzik for $\lambda_{(1)} = -(3/2)\,16\pi^2 = -24\pi^2$ implying due to $\lambda_{(1)} = -g^4_{(1)}/(32\pi^2 m^4_{(1)})$ four solutions for the three-point coupling constant $g_{(1)}$, i.e., $g_{(1)} = \pm 4\pi\, 3^{1/4}\, m_{(1)}$ and $g_{(1)} = \pm\mathrm{i}\, 4\pi\, 3^{1/4}\, m_{(1)}$. Furthermore we notice that for the probable case of $C \neq -4$ and non-vanishing mass $m_{(1)}$ the non-trivial theory develops already at this stage a finite non-vanishing vacuum expectation value (see also the discussion in Ref. [23]). Finally we mention in view of self-consistency without listing the explicit proof that the obtained non-trivial values for $\lambda_{(1)}$ and $g_{(1)}$ lead also to a cancellation of quadratic divergencies on the level of the selfenergy, consistent with our starting assumption that quadratic divergencies cancel.

2.2 A non-Hermitian and "PT-symmetric" theory of strong interactions

The purpose of this section is to demonstrate on the basis of experimental "evidence" that a dynamically generated theory of strong interactions based on mesons and quarks has to be non-Hermitian and close to PT-symmetric [6]. Starting point for our considerations — inspired somehow by Ref. [24] — is the sum of the interaction Lagrangean of weak interactions containing (anti)leptons denoted by $\ell_-(x)$, $\overline{\ell^c_+}(x)$ and (anti)quarks denoted by $q_-(z)$, $\overline{q^c_+}(z)$ and a Yukawa-like interaction Lagrangean describing the strong interaction between (anti)quarks and scalar ($S(z)$), pseudoscalar ($P(z)$), vector ($V(z)$), and axial vector ($Y(z)$) $U(6)\times U(6)$ meson field

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$$= -2\frac{\mathrm{i}}{16\pi^2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{(p^2-m^2)} + \frac{2}{3}\, m^2 \left(\frac{3}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{(p^2-m^2)^2} + \frac{\mathrm{i}}{16\,\pi^2}\right)^2 + \left(\frac{1}{16\pi^2}\right)^2 m^2 \left(\frac{1}{6} - C\right) + \ldots \,. \tag{16}$$

The last line displays the most divergent part of the massive sunset/sunrise diagram at zero external four-momentum in a regularization scheme independent manner. The application of a renormalization scheme yielding the "bootstrapping" log.-divergent gap-equation Eq. (11) finally reduces the foregoing equation to

$$I_{\text{sunset/sunrise}} \to -2\,\frac{\mathrm{i}}{16\pi^2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\frac{1}{(p^2-m^2)} - \left(\frac{1}{16\pi^2}\right)^2 m^2\,(4+C) + \ldots \,. \tag{17}$$

matrices in flavour space inspired by Ref. [25] (see also [29, 30]) (the undetermined signs s_s, s_p, s_v, $s_y \in \{-1,+1\}$ are here irrelevant!):

$$\mathcal{L}^{\rm strong}_{\rm int}(z) = \\ = \sqrt{2} g\, \overline{q^c_+}(z) \Big(s_s S(z) + s_p {\rm i} P(z)\gamma_5 + \frac{{\rm e}^{-{\rm i}\alpha}}{2}\Big(s_v \not{V}(z) + s_y \not{Y}(z)\gamma_5 \Big)\Big)\, q_-(z), \quad (19)$$

with $g = |g|\exp({\rm i}\alpha)$ being the eventually complex strong interaction coupling constant, while contrary to Refs. [24, 25] we *do not allow any further extra direct meson-meson interaction terms* in the Lagrangean, as they shall be dynamically generated through quark-loops only [7]). The first step is now to study leptonic decays of pseudoscalar mesons to extract the pseudoscalar decay constants f_P. By dynamical generation we obtain for the relevant part of the effective action $S_{\rm eff}$ in the local limit ($M_q \equiv {\rm diag}[m_u, m_c, m_t, m_d, m_s, m_b]$, "tr$_{\rm F}$"= flavour trace) [8]):

$$\begin{aligned}
\frac{\rm i}{1!}\mathcal{S}_{\rm eff} &= \frac{\rm i}{1!}\,\langle 0|T[S\,]\,|0\rangle_{\rm c}\Big|_{{\rm except}\ P\bar{\ell}\ell} + \frac{{\rm i}^2}{2!}\,\langle 0|T[\mathcal{S}\,\mathcal{S}\,]\,|0\rangle_{\rm c}\Big|_{{\rm except}\ P\bar{\ell}\ell} + \ldots \\
&= \int {\rm d}^4 z\, \left(-2\frac{G_{\rm F}}{\sqrt{2}}\right)\sqrt{2} s_p {\rm e}^{{\rm i}\alpha} \\
&\quad \times {\rm tr}_{\rm F}\Big[-4\,{\rm i}\, N_{\rm c}|g| \int \frac{{\rm d}^4 p}{(2\pi)^4}\frac{1}{p^2 - M_q^2}\frac{1}{2}\{M_q\,,(\partial_\mu P(z))\}\frac{1}{p^2-M_q^2} \\
&\quad \times \Big(\overline{\ell^c_+}(z)\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma_5)\begin{pmatrix} 0_3 & 0_3 \\ 1_3 & 0_3\end{pmatrix}\ell_-(z)\left[\begin{pmatrix} 0_3 & V_{\rm CKM} \\ 0_3 & 0_3\end{pmatrix}\right] \\
&\quad + \overline{\ell^c_+}(z)\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma_5)\begin{pmatrix} 0_3 & 1_3 \\ 0_3 & 0_3\end{pmatrix}\ell_-(z)\left[\begin{pmatrix} 0_3 & 0_3 \\ \overline{V}_{\rm CKM} & 0_3\end{pmatrix}\right] \\
&\quad + \overline{\ell^c_+}(x)\,\gamma^\mu\,\frac{1}{2}\Big(T_3\,(1-\gamma_5) - 2\,Q_\ell\,\sin^2\theta_W\Big)\,\ell_-(z)\,\Big[\,2\,T_3\Big]\Big)\Big] + \ldots . \quad (20)
\end{aligned}$$

Inspection yields for the decay constant $f_{\eta_{q_1\bar{q}_2}}$ of a pseudoscalar meson $\eta_{q_1\bar{q}_2}$

$$ {\rm i} f_{\eta_{q_1\bar{q}_2}} \longleftrightarrow 4N_{\rm c}|g| \int \frac{{\rm d}^4 p}{(2\pi)^4}\frac{\frac{1}{2}(m_{q_1}+m_{\bar{q}_2})}{(p^2-m^2_{q_1})(p^2-m^2_{\bar{q}_2})}\,, \quad (21)$$

being in accordance with the log.-divergent gap-equation Eq. (11) promoted by M.D. Scadron [9]). As we shall need it in the following, we have now to dynamically

[7]) This follows the same philosophy as in the previous section, where the ϕ^4-interaction was dynamically generated starting out just from a ϕ^3-theory. It is an interesting possibility to be considered in future, whether in a similar manner the whole non-fermionic part of the Lagrangean of the standard model of particle physics can be dynamically generated on the basis of Yukawa-like interaction terms coupling bosons (gauge-bosons, Higgs-(pseudo)scalars, ...) to fermions, i.e. (anti)quarks and (anti)leptons.

[8]) We assumed here without loss of generality for traditional reasons a colour factor $N_{\rm c}$, which can be absorbed by a redefinition of the strong coupling constant g.

[9]) The log.-divergent gap-equation should be understood here as a prescription to renormalize the original unrenormalized Lagrangean in replacing originally divergent quantitites by finite experimental numbers through a suitable choice of counter terms implying Eq. (11). It is interesting to note that the previous result yields the extremly important sum-rule (resulting from the properties of the underlying integral) $(m_{q_1} - m_{\bar{q}_3})\, f_{\eta_{q_1\bar{q}_3}} = (m_{q_1} - m_{\bar{q}_2})\, f_{\eta_{q_1\bar{q}_2}} + (m_{q_2} - m_{\bar{q}_3})\, f_{\eta_{q_2\bar{q}_3}}$ yielding e.g. $(m_u - m_s)\, f_{\rm K^+} = (m_u - m_d)\, f_{\pi^+} + (m_d - m_s)\, f_{\rm K^0}$.

generate the effective action describing the coupling of a scalar and two pseudoscalar mesons. The result is listed in the footnote [10]). In order to arrive at our final conclusions, we can use the previous result to study the experimentally measured transition formfactors $f_{\pm}^{\mathrm{K}^+\pi^0}(0)$ characterizing the process $\mathrm{K}^+ \to \pi^0\, \mathrm{e}^+ \nu_{\mathrm{e}}$ at zero four-momentum transfer. First we dynamically generate the respective effective action in the local limit displaying here only the terms relevant for us representing W-emission graphs and an exchange of a scalar κ^+-meson due to Partial Conservation of Vector Currents (PCVC)[26] [11]):

$$\begin{aligned}\mathcal{S}_{\mathrm{eff}} \;=\; & \int \mathrm{d}^4 z\, (-\mathrm{i}\, \mathrm{e}^{2\,\mathrm{i}\,\alpha}) \left(-\frac{G_{\mathrm{F}}}{\sqrt{2}}\right) \overline{V}_{us}\;\; \overline{e^{c}_{+}}\,(z)\, \gamma_\mu\, (1-\gamma_5)\, \nu_{e\,-}(z) \\ & \times \frac{1}{\sqrt{2}} \left\{ \pi^0(z) \left(\frac{2\,|\,g|\, f_{\mathrm{K}^+}}{m_u + m_s}\; \partial^\mu K^+(z) \right) - K^+(z) \left(\frac{2\,|\,g|\, f_{\eta_{u\bar u}}}{m_u + m_u}\; \partial^\mu \pi^0(z) \right) \right. \\ & + 4\,\mathrm{i}\, N_{\mathrm{c}}\, |\,g|^2\, (m_s - m_u)^2\; K^+(z)\, (\partial^\mu \pi^0(z)) \int \frac{\mathrm{d}^4 p}{(2\,\pi)^4}\, \frac{1}{(p^2 - m_s^2)(p^2 - m_u^2)^2} \\ & \left. + \frac{\lambda}{g^2}\, m_s\, \frac{(m_s - m_u)}{m^2_{\kappa^+}}\, \frac{2\,|\,g|\, f_{\mathrm{K}^+}}{m_u + m_s} \Big(\pi^0(z)\, (\partial^\mu K^+(z)) + K^+(z)\, (\partial^\mu \pi^0(z)) \Big) \right\} \\ & + \mathrm{K}^*\text{-exchange} + \ldots\,. \end{aligned} \tag{23}$$

From this result we can read off the desired transition formfactors $f_{\pm}^{\mathrm{K}^+\pi^0}(0)$ at zero four momentum transfer. Displaying only terms being of relevant order in the scale $\delta = (m_s/m_u) - 1 \simeq 0.44$ according to the nonrenormalization theorem of M. Ademollo and R. Gatto [27], we obtain $f_{+}^{\mathrm{K}^+\pi^0}(0) = 1 + O(\delta^2)$ and

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[10]) In the considered local limit we obtain:

$$\begin{aligned} \frac{\mathrm{i}}{1!}\, \mathcal{S}_{\mathrm{eff}} = \frac{\mathrm{i}^3}{3!}\, \frac{3!}{1!\,2!}\, \langle 0|\, T[\, \mathcal{S}^{Sq\bar q}_{\mathrm{int}}\, \mathcal{S}^{Pq\bar q}_{\mathrm{int}}\, \mathcal{S}^{Pq\bar q}_{\mathrm{int}}\,]\, |0\rangle_{\mathrm{c}}\, \Big|_{\mathrm{except}\ SPP} \\ \overset{!}{=} \int \mathrm{d}^4 z \sqrt{2}\, g^2\, \mathrm{e}^{\mathrm{i}\,\alpha} s_s (-4\,\mathrm{i}\, N_{\mathrm{c}}\, |g|) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \Big\{ \mathrm{tr_F} \Big[S(z) \frac{1}{(p^2 - M_q^2)} \{P^2(z), M_q\} \frac{1}{(p^2 - M_q^2)} \Big] \\ + \mathrm{tr_F} \Big[\, [\, S(z)\,,\, P(z)]\; \frac{1}{(p^2 - M_q^2)}\; [\, P(z)\,,\, M_q\,]\; \frac{1}{(p^2 - M_q^2)}\; \Big] \\ - \mathrm{tr_F} \Big[\{S(z), M_q\} \frac{1}{(p^2 - M_q^2)} [P(z), M_q] \frac{1}{(p^2 - M_q^2)} [P(z), M_q\,] \frac{1}{(p^2 - M_q^2)} \Big] \Big\} + \ldots\,. \end{aligned} \tag{22}$$

Recalling our "defining" equation for pseudoscalar decay constants, Eq. (21), the first two terms on the right-hand side of Eq. (22) are equivalent to an SPP-interaction term, which one would obtain from a "shifted" quartic interaction Lagrangean with quartic coupling λ. The "shifted" Lagrangean is $\mathcal{L}(x) = -(\lambda/2)\, \mathrm{tr_F}[((S(x) + \mathrm{i}\, P(x) - D)(S(x) - \mathrm{i}\, P(x) - D))^2] = \lambda\; \mathrm{tr_F}[\,(S(x) + \mathrm{i}\, P(x))(S(x) - \mathrm{i}\, P(x))(\{S(x), D\} + \mathrm{i}\, [P(x), D])] + \ldots$. The quantity D is the matrix (identified with decay constants of neutral pseudoscalar mesons) leading to spontaneous symmetry breaking according to the shift $S(x) \to S(x) - D$ and inducing quark-masses according to the relation $M_q = \sqrt{2}\; g\; s_s\; D$. The last term on the right-hand side of Eq. (22) involving only commutators $[\,P(z)\,,\, M_q\,]$ is proportional to the square of quark-mass differences and therefore small in the sense of t he non-renormalization theorem by M. Ademollo and R. Gatto [27].

[11]) The exchange of a vector meson K^* is here disregarded, as it can contribute only marginally to the transition formfactor $f_{+}^{\mathrm{K}^+\pi^0}(0)$ at zero four-momentum transfer, i.e. at most of the order of the non-renormalization theorem by M. Ademollo and R. Gatto [27], as the charge of the K^+ is solely generated due to photon–quark interactions.

$$f_-^{K^+\pi^0}(0) - O(\delta^2) = \frac{\lambda}{g^2}\, m_s\, \frac{(m_s - m_u)}{m_{\kappa^+}^2}\, \frac{2\,|g|\, f_{K^+}}{m_u + m_s}$$
$$= e^{2i\alpha}\, \frac{\lambda}{g^2}\, \frac{2\,\delta\,(1+\delta)}{(2+\delta)}\, \frac{|m_u||f_{K^+}|}{m_{\kappa^+}^2}\, |g| \stackrel{!}{=} e^{2i\alpha}\, \frac{2\,\delta\,(1+\delta)}{(2+\delta)}\, \frac{|m_u||f_{K^+}|}{m_{\kappa^+}^2}\, \frac{4\pi}{\sqrt{3}}\,. \quad (24)$$

On the right-hand side of this equation we used that M.D. Scadron's log.-div. gap-equation Eq. (11) in combination with Eq. (21) implies $|g| = 2\pi/\sqrt{N_c} = 2\pi/\sqrt{3}$, and that there holds $\lambda \simeq 2\,g^2$ according to a one-loop dynamical generation [1]. In using the experimental values [28] $|f_{K^+}| \simeq 159$ MeV$/\sqrt{2}$ and $m_{\kappa^+} \simeq 797$ MeV we produce with the help of the last line of Eq. (24) the following table:

$f_-^{K^+\pi^0}(0)/e^{2i\alpha}$	0.050	0.102	0.125	0.148	0.200	0.225
δ for $\|m_u\| = 337$ MeV	0.1098	0.2149	0.2591	0.3023	0.3965	0.4404
δ for $\|m_u\| = 3$ MeV	7.274	14.11	17.12	20.12	26.89	30.14

Inspection of the constituent quark mass case $|m_u| \simeq 337$ MeV reveils that the experimentally measured *negative* transition formfactor ratio $f_-^{K^+\pi^0}(0)/f_+^{K^+\pi^0}(0) \simeq -0.125 \pm 0.023$ [28] can be only accomodated for $e^{2i\alpha} < 0$, while for reasonable values of δ and m_κ experiment seems to suggest the extreme PT-symmetric [6] case $\alpha \simeq -\pi/2+0$, yielding an imaginary PT-symmetric Yukawa-coupling $g = -i\,2\pi/\sqrt{3}$ and a Symanzik-like quartic coupling $\lambda \simeq 2\,g^2 = -8\pi^2/3 < 0$, as obtained already earlier by the author, when "deriving" the Lagrangean of the Quark-Level Linear Sigma Model from the Lagrangean of QCD [29] (see also Ref. [30]). Finally it is interesting to consider our rough estimate for the experimentally yet badly determined mass of the $\kappa(800)$ scalar resonance (biased by $K_0^*(1430)$) as a function of δ. For $|m_u| \simeq 337$ MeV and $f_-^{K^+\pi^0}(0) = -0.125$ we obtain the following table:

δ	0.10	0.20	0.26	0.30	0.40	0.44	0.50
m_κ (MeV)	480.0	692.7	798.5	863.6	1013.1	1068.69	1148.7

Hence, semileptonic decays of pseudoscalar mesons cannot only be used to reveal the seemingly non-Hermitian nature of a theory of strong interaction with a sizable amount of scalar confinement, they also may be used to "measure" badly known experimental quantities like the masses of light and heavy scalar resonances.

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