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$SU(3)$ Mass Splittings for $\bar{q}q$ Mesons and qqq Baryons

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Abstract. By comparing $SU(3)$-breaking scales of linear mass formulae, it is shown that the lowest vector, axial-vector, and scalar mesons all have a $\bar{q}q$ configuration, while the ground-state octet and decuplet baryons are qqq. Also, the quark-level linear σ model is employed to predict similar $\bar{q}q$ and qqq states. Finally, the approximate mass degeneracy of the scalar $a_0(980)$ and $f_0(980)$ mesons is demonstrated to be accidental.

1. INTRODUCTION

In the quark model, one usually assumes that pseudoscalar ($\mathscr{P}$), vector ($\mathscr{V}$), and axial-vector ($\mathscr{A}$) mesons are $\bar{q}q$, whereas octet ($\mathscr{O}$) and decuplet ($\mathscr{D}$) baryons are qqq states. However, it is often argued [1] that the light scalar ($\mathscr{S}$) mesons are non-$\bar{q}q$ candidates, in view of their low masses. In this short paper, we shall show that the ground-state meson nonets $\mathscr{P}$, $\mathscr{S}$, $\mathscr{V}$, and $\mathscr{A}$ are *all* $\bar{q}q$, hence including the light scalars, while the lowest $\mathscr{O}$ and $\mathscr{D}$ baryons are qqq states.

In Sec. 2, $SU(3)$ mass splittings for loosely bound $\mathscr{V}$, $\mathscr{A}$, and $\mathscr{S}$ states are shown to have symmetry-breaking scales of 16%, 8%, and 25%, respectively, using linear mass formulae. We apply the latter formulae to qqq $\mathscr{O}$ and $\mathscr{D}$ states in Sec. 3, leading to $SU(3)$-breaking scales of 13% and 12%, respectively. Then in Sec. 4, we employ the quark-level linear σ model (LσM) to predict similar $\bar{q}q$ and qqq states as in Secs. 2 and 3. Next in Sec. 5, we study the $\mathscr{S}$ $\bar{q}q$ states and argue why the $\mathscr{V}$ states have slightly higher masses, on the basis of the nonrelativistic quark model. Moreover, the approximate mass degenaracy of the $\mathscr{S}$ $a_0(980)$ and $f_0(980)$ mesons is shown to be just accidental. We summarize our results in Sec. 6.

2. MASS SPLITTINGS FOR $SU(3)$ $\mathscr{V}$, $\mathscr{S}$, $\mathscr{A}$ GROUND STATES

Although meson masses are expected to appear *quadratically* in model Lagrangians, while they must appear so for $\mathscr{P}$ states [2], for $\mathscr{V}$, $\mathscr{S}$, $\mathscr{A}$ states a Taylor-series linear form for $SU(3)$ mass splittings is also possible. Thus consider a Hamiltonian density $H = H(\lambda_0) + H_{ss}(\lambda_8)$ using Gell-Mann matrices. Then the vector-meson-nonet masses

$m_{\mathscr{V}} = m^0_{\mathscr{V}} - \delta m_{\mathscr{V}}\, d_{\bar{i}8i}$ are

$$\begin{aligned} m_{\rho,\omega} &= m^0_{\mathscr{V}} - \frac{\delta m_{\mathscr{V}}}{\sqrt{3}} \approx 776\ \text{MeV}\,, \\ m_{K^*} &= m^0_{\mathscr{V}} + \frac{\delta m_{\mathscr{V}}}{2\sqrt{3}} \approx 892\ \text{MeV}\,, \\ m_{\phi} &= m^0_{\mathscr{V}} + \frac{2\delta m_{\mathscr{V}}}{\sqrt{3}} \approx 1020\ \text{MeV}\,, \end{aligned} \tag{1}$$

with $\phi \approx \bar{s}s$. Measured vector masses [1] suggest average mass splittings

$$m^0_{\mathscr{V}} \approx 850\ \text{MeV} \quad , \quad \delta m_{\mathscr{V}} \approx 140\ \text{MeV} \quad , \tag{2}$$

giving an $SU(3)$-breaking scale of $\delta m_{\mathscr{V}}/m^0_{\mathscr{V}} \approx 16\%$.

Likewise, the ground-state axial-vector mesons (while slightly ambiguous) still suggest

$$\begin{aligned} m_{a_1(1260),\, f_1(1285)} &= m^0_{\mathscr{A}} - \frac{\delta m_{\mathscr{A}}}{\sqrt{3}} \approx 1256\ \text{MeV}\,, \\ m_{K_1(1270)} &= m^0_{\mathscr{A}} + \frac{\delta m_{\mathscr{A}}}{2\sqrt{3}} \approx 1273\ \text{MeV}\,, \\ m_{f_1(1420)} &= m^0_{\mathscr{A}} + \frac{2\delta m_{\mathscr{A}}}{\sqrt{3}} \approx 1426\ \text{MeV}\,. \end{aligned} \tag{3}$$

Here, we assume the $f_1(1420)$ is mostly $\bar{s}s$, because the PDG [1] reports $f_1(1420) \to KK\pi, K^*K$ as dominant, while $f_1(1285) \to KK\pi, K^*K$ are almost absent. Thus, $f_1(1285)$ is mostly $\bar{n}n$, like the nonstrange $a_1(1260)$ (with $a_1 \to \sigma\pi$ seen, but $a_1 \to f_0(980)\pi$ not seen, because $f_0(980)$ is mostly $\bar{s}s$). Then the pattern of Eqs. (3) suggests approximate average mass splittings

$$m^0_{\mathscr{A}} \approx 1305\ \text{MeV} \quad , \quad \delta m_{\mathscr{A}} \approx 98\ \text{MeV} \quad , \quad \frac{\delta m_{\mathscr{A}}}{m^0_{\mathscr{A}}} \approx 8\% \quad . \tag{4}$$

Also the scalar masses (not incompatible with Ref. [1]) predicted from the LσM discussed in Sec. 4 obey the mass-splitting pattern

$$\begin{aligned} m_{\sigma_n} &= m^0_{\mathscr{S}} - \frac{\delta m_{\mathscr{S}}}{\sqrt{3}} \approx 650\ \text{MeV}\,, \\ m_{\kappa} &= m^0_{\mathscr{S}} + \frac{\delta m_{\mathscr{S}}}{2\sqrt{3}} \approx 800\ \text{MeV}\,, \\ m_{\sigma_s} &= m^0_{\mathscr{S}} + \frac{2\delta m_{\mathscr{S}}}{\sqrt{3}} \approx 970\ \text{MeV}\,. \end{aligned} \tag{5}$$

Here, $m_{\sigma_n(650)}$ is near the PDG average [1] $m_{f_0(600)}$, $m_{\kappa(800)}$ is near the E791 value [3] 797 ± 19 MeV, and $m_{\sigma_s(970)}$ from Sec. 5 and the Appendix is near the PDG value

$m_{f_0(980)}$, which is thus mostly $\bar{s}s$. The masses from Eqs. (5) then give the average mass splittings

$$m^0_{\mathscr{S}} \approx 753\,\text{MeV} \quad , \quad \delta m_{\mathscr{S}} \approx 185\,\text{MeV} \quad , \quad \frac{\delta m_{\mathscr{S}}}{m^0_{\mathscr{S}}} \approx 25\% \quad . \tag{6}$$

The fact that the $\bar{q}q$ scalars have an $SU(3)$-breaking scale of 25%, about double the scale of $\mathscr{V}$ and $\mathscr{A}$ ground states, further suggests that, whereas the $\mathscr{V}$, $\mathscr{A}$ are $\bar{q}q$ loosely bound states, the $\bar{q}q$ $\mathscr{S}$ states (with quarks touching in the NJL scheme [4]) are "barely" elementary-particle partners of the tightly bound $\mathscr{P}$ states (discussed in Sec. 4).

The mean of the slightly varying $\mathscr{V}$, $\mathscr{A}$, $\mathscr{S}$ mass scales in Eqs. (2,4,6) is $m^0 = 969$ MeV, $\delta m = 141$ MeV, and the latter are close to the baryon mass-splitting scales which we derive next.

3. LOOSELY BOUND QQQ BARYONS

In this same Taylor-series spirit, the $\mathscr{O}$ baryon $SU(3)$ mass splitting $m_{\mathscr{O}} = m^0_{\mathscr{O}} - \delta m_{\mathscr{O}}(d_{ss}d^{\bar{i}8i} + f_{ss}if^{\bar{i}8i})$ for $d_{ss} + f_{ss} = 1$, predicts (the index ss means semistrong)

$$\begin{aligned}
m_N &= m^0_{\mathscr{O}} - \frac{\delta m_{\mathscr{O}}}{2\sqrt{3}}(-d_{ss} + 3f_{ss}) \approx 939\,\text{MeV}\,, \\
m_\Lambda &= m^0_{\mathscr{O}} + \frac{\delta m_{\mathscr{O}}}{\sqrt{3}} d_{ss} \approx 1116\,\text{MeV}\,, \\
m_\Sigma &= m^0_{\mathscr{O}} - \frac{\delta m_{\mathscr{O}}}{\sqrt{3}} d_{ss} \approx 1193\,\text{MeV}\,, \\
m_\Xi &= m^0_{\mathscr{O}} + \frac{\delta m_{\mathscr{O}}}{2\sqrt{3}}(d_{ss} + 3f_{ss}) \approx 1318\,\text{MeV}\,.
\end{aligned} \tag{7}$$

The $(d/f)_{ss}$ ratio can be found from Eqs. (7) as

$$\left(\frac{d}{f}\right)_{ss} = -\frac{3}{2}\frac{m_\Sigma - m_\Lambda}{m_\Xi - m_N} \approx -0.305 \quad , \quad d_{ss} \approx -0.44 \quad , \quad f_{ss} \approx 1.44 \quad . \tag{8}$$

Thus, Eqs. (7) predict the average mass splittings

$$m^0_{\mathscr{O}} \approx 1151\,\text{MeV} \quad , \quad \delta m_{\mathscr{O}} \approx 150\,\text{MeV} \quad , \quad \frac{\delta m_{\mathscr{O}}}{m^0_{\mathscr{O}}} \approx 13\% \quad . \tag{9}$$

The $SU(3)$ $\mathscr{D}$ baryon masses $m_{\mathscr{D}} = m^0_{\mathscr{D}} + \delta m_{\mathscr{D}}$ have $m^0_{\mathscr{D}}$ weighted by wave functions

$$\overline{\Psi}^{(abc)}\Psi_{(abc)} = \overline{\Delta}\Delta + \overline{\Sigma}^*\Sigma^* + \overline{\Xi}^*\Xi^* + \overline{\Omega}\Omega\,, \tag{10}$$

and $\delta m_{\mathscr{D}}$ is weighted by

$$3\overline{\Psi}^{(ab3)}\Psi_{(ab3)} = \overline{\Sigma}^*\Sigma^* + 2\overline{\Xi}^*\Xi^* + 3\overline{\Omega}\Omega\,. \tag{11}$$

Then the $SU(3)$ $\mathscr{D}$ masses are predicted (in MeV) to be

$$\begin{aligned} m_{\Delta} &= m^0_{\mathscr{D}} &&\approx 1232\,, \\ m_{\Sigma^*} &= m^0_{\mathscr{D}} + \delta m_{\mathscr{D}} &&\approx 1385\,, \quad \text{with } \delta m_{\mathscr{D}} \approx 153\,, \\ m_{\Xi^*} &= m^0_{\mathscr{D}} + 2\delta m_{\mathscr{D}} &&\approx 1533\,, \quad \text{with } \delta m_{\mathscr{D}} \approx 151\,, \\ m_{\Omega} &= m^0_{\mathscr{D}} + 3\delta m_{\mathscr{D}} &&\approx 1672\,, \quad \text{with } \delta m_{\mathscr{D}} \approx 147\,. \end{aligned} \tag{12}$$

This corresponds to average mass splittings

$$m^0_{\mathscr{D}} \approx 1232 \text{ MeV} \quad , \quad \delta m_{\mathscr{D}} \approx 150 \text{ MeV} \quad , \quad \frac{\delta m_{\mathscr{D}}}{m^0_{\mathscr{D}}} \approx 12\% \quad . \tag{13}$$

It is interesting that both loosely bound qqq $\mathscr{O}$ and $\mathscr{D}$ symmetry-breaking scales of about 150 MeV are near the $\bar{q}q$ $\mathscr{V}$, $\mathscr{A}$, $\mathscr{S}$ mean mass-splitting scale of $\delta m = 141$ MeV. However, the $SU(3)$-breaking scale of 25% for scalars is almost double the 12–16% scales of $\mathscr{V}$, $\mathscr{A}$, $\mathscr{O}$, $\mathscr{D}$ states. This suggests that $\mathscr{V}$, $\mathscr{A}$, $\mathscr{O}$, $\mathscr{D}$ $\bar{q}q$ or qqq states are all loosely bound, in contrast with the $\bar{q}q$ $\mathscr{S}$ and, of course, the $\mathscr{P}$ states (see above). In fact, the latter Nambu–Goldstone $\mathscr{P}$ states are massless in the chiral limit (CL) $p^2 = m^2_{\pi} = 0$, $p^2 = m^2_K = 0$, as the tightly-bound measured [1] π^+ and K^+ charge radii indicate [5].

4. CONSTITUENT QUARKS AND THE QUARK-LEVEL LσM

Formulating the $\mathscr{P}$ and $\mathscr{S}$ $\bar{q}q$ states as elementary chiral partners [6], the Lagrangian density of the $SU(2)$ quark-level linear σ model (LσM) has, after the spontaneous-symmetry-breaking shift, the interacting part [7]

$$\mathscr{L}^{\text{int}}_{\text{L}\sigma\text{M}} = g\,\bar{\psi}(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})\psi + g'\,\sigma\,(\sigma^2+\pi^2) - \frac{\lambda}{4}(\sigma^2+\pi^2)^2 - f_\pi g\,\bar{\psi}\psi\,, \tag{14}$$

with tree-order CL couplings related as (for $f_\pi \approx 93$ MeV)

$$g = \frac{m_q}{f_\pi} \quad , \quad g' = \frac{m^2_\sigma}{2f_\pi} = \lambda\, f_\pi\,. \tag{15}$$

The $SU(2)$ and $SU(3)$ chiral Goldberger–Treiman relations (GTRs) are

$$f_\pi\, g = \hat{m} = \frac{1}{2}(m_u + m_d) \quad , \quad f_K\, g = \frac{1}{2}(m_s + \hat{m})\,. \tag{16}$$

Since $f_K/f_\pi \approx 1.22$ [1], the constituent-quark-mass ratio from Eq. (16) becomes

$$1.22 \approx \frac{f_K}{f_\pi} = \frac{1}{2}\left(1 + \frac{m_s}{\hat{m}}\right) \quad \Rightarrow \quad \frac{m_s}{\hat{m}} \approx 1.44\,, \tag{17}$$

which is independent of the value of g. In loop order, Eqs. (15) are recovered, along with [8, 5]

$$m_\sigma = 2m_q \quad , \quad g = \frac{2\pi}{\sqrt{N_c}} \quad , \quad \text{for } N_c = 3\,. \tag{18}$$

Here, the first equation is the NJL relation [4], now true for the LσM as well. The second equation in Eq. (18) was first found via the $Z = 0$ compositeness relation [9], separating the elementary π and σ particles from the bound states ρ, ω, and a_1.

We first estimate the (non-chiral-limiting) nonstrange and strange constituent quark masses from the GTRs (16), together with the LσM loop-order result (18):

$$\begin{array}{rcccccl} \hat{m} & \approx & g f_\pi & \approx & \dfrac{2\pi}{\sqrt{3}}(93\,\text{MeV}) & \approx & 337\,\text{MeV}\,, \\ m_s & = & \left(\dfrac{m_s}{\hat{m}}\right)\hat{m} & \approx & 1.44\,\hat{m} & \approx & 485\,\text{MeV}\,. \end{array} \tag{19}$$

These quark-mass scales in turn confirm the mass-splitting scales found in Secs. 2, 3:

$$\delta m_{\mathscr{V}} = \delta m_{\mathscr{A}} = \delta m_{\mathscr{O}} = \delta m_{\mathscr{D}} \approx (485 - 337)\,\text{MeV} = 148\,\text{MeV}\,, \tag{20}$$

near 140, 98, 150, and 150 MeV, respectively. Also the $SU(3)$ non-vanishing masses are predicted as

$$\begin{array}{rcccccl} m^0_{\mathscr{V}} & = & m^0_{\mathscr{A}} & \approx & m_s + \hat{m} & \approx & 822\,\text{MeV}\,, \\ m^0_{\mathscr{O}} & = & m^0_{\mathscr{D}} & \approx & m_s + 2\hat{m} & \approx & 1160\,\text{MeV}\,, \end{array} \tag{21}$$

near the 850, 1151, and 1232 MeV m^0 masses in Secs. 2, 3.

5. $\mathscr{S}$ SCALARS AND ACCIDENTAL DEGENERACIES

An almost degenerate case in the nonrelativistic quark model (NRQM) is [10], in the context of QCD,

$$m_{\mathscr{S}} \approx m_{\mathscr{V}} - \frac{2\alpha_s^{\text{eff}}}{m^2_{\text{dyn}}}\left(\frac{\vec{L}\cdot\vec{S}}{r^3}\right) = 780\,\text{MeV} - 140\,\text{MeV} = 640\,\text{MeV}\,, \tag{22}$$

where the ground-state vector mesons have $L = 0$ and so no spin-orbit contribution to the mass. This corresponds to $m_{\sigma(650)} \approx m_{\omega(782)} - 140$ MeV $= 642$ MeV. Equivalently, invoking the $I = 1/2$ CGC of $1/2$, one predicts via the NRQM $m_{\kappa(800)} \approx m_{K^*(892)} - 70$ MeV $= 822$ MeV. Or invoking instead the $\bar{s}s$ CGC of $1/3$, one gets $m_{\sigma_s(970)} \approx m_{\phi(1020)} - 47$ MeV $= 973$ MeV.

However, for the elementary-particle $\mathscr{P}$ and $\mathscr{S}$ states, one should invoke the infinite-momentum-frame (IMF, see Appendix) scalar-pseudoscalar $SU(3)$ equal-splitting laws (ESLs), reading [11]

$$m^2_\sigma - m^2_\pi \approx m^2_\kappa - m^2_K \approx m^2_{a_0} - m^2_{\eta\text{avg}} \approx 0.40\,\text{GeV}^2\,, \tag{23}$$

where $m_{\eta\text{avg}}$ is the average η, η' mass 753 MeV. These ESLs hold for $m_{\sigma(650)} = 2\hat{m}$ and $m_{\kappa(800)} = 2\sqrt{m_s\hat{m}} = 809$ MeV, the NJL-LσM values. Using the ESLs (23) to predict the a_0 mass, one finds

$$m_{a_0} = \sqrt{0.40\,\text{GeV}^2 + m^2_{\eta\text{avg}}} \approx 983.4\,\text{MeV}\,, \tag{24}$$

very close to the PDG value 984.7 ± 1.2 MeV. Thus, the nearness of the $a_0(980)$ and $f_0(980)$ masses, the latter scalar being mostly $\bar{s}s$ and so near the vector $\bar{s}s$ $\phi(1020)$ (see above), is indeed an accidental degeneracy. Note that a similar (approximate) degeneracy is found in the dynamical unitarized quark-meson model of Ref. [12], where the same $\bar{q}q$ assignments are employed as here.

6. SUMMARY AND CONCLUSIONS

The usual field-theory picture is that meson masses should appear quadratically and baryon masses linearly in Lagrangian models based on the Klein–Gordon and Dirac equations. However, in Secs. 2 and 3 we have studied both mesons and baryons in a *linear*-mass $SU(3)$-symmetry Taylor-series sense. Instead, in Sec. 5 we have studied symmetry breaking in the IMF, with $E = [p^2 + m^2]^{1/2} \approx p\,[1 + m^2/2p^2 + \ldots]$. Here, between brackets, the 1 indicates the symmetry limit, and the *quadratic* mass term means that both meson and baryon masses are *squared* in the mass-breaking IMF for $\Delta S = 1$ ESLs. While the former mass-splitting approach (with linear masses) fits all $\mathscr{V}$, $\mathscr{A}$, $\mathscr{S}$, $\mathscr{O}$, and $\mathscr{D}$ ground-state $SU(3)$-flavor multiplets, so does the latter (with quadratic masses) for the IMF-ESLs. Nevertheless, Nambu–Goldstone pseudoscalars $\mathscr{P}$ *always* involve *quadratic* masses. Both approaches suggest that all ground-state mesons ($\mathscr{P}$, $\mathscr{S}$, $\mathscr{V}$, $\mathscr{A}$) are $\bar{q}q$ states, while baryons ($\mathscr{O}$, $\mathscr{D}$) are qqq states. This picture is manifest in the quark-level LσM of Sec. 4. Finally, the accidental scalar degeneracy between the $\bar{s}s$ $f_0(980)$ and the $\bar{n}n$ $a_0(980)$ was explained in Sec. 5, via the IMF quadratic-mass ESLs — also compatible with mesons being $\bar{q}q$ and baryons qqq states.

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A. KINEMATIC INFINITE-MOMENTUM FRAME

The infinite-momentum frame (IMF) has two virtues: (i) $E = [p^2 + m^2]^{1/2} \approx p + m^2/2p + \ldots$, for $p \to \infty$, requires *squared* masses when the lead term p is eliminated, using $SU(3)$ formulae with coefficients $1+3 = 2+2$, as e.g. the Gell-Mann–Okubo linear mass formula $\Sigma+3\Lambda = 2N+2\Xi$, valid to 3%; (ii) when $p \to \infty$, dynamical tadpole graphs are suppressed [13]. In fact, $\Sigma^2+3\Lambda^2 = 2N^2+2\Xi^2$ is also valid empirically to 3%. This squared qqq baryon mass formula can be interpreted as a $\Delta S = 1$ ESL, which holds for

both $\mathscr{O}$ and $\mathscr{D}$ baryons [11]:

$$\begin{aligned} \Sigma\Lambda - N^2 \;&\approx\; \Xi^2 - \Sigma\Lambda \;\approx\; \frac{1}{2}\left(\Xi^2 - N^2\right) &&\approx\; 0.43\,\mathrm{GeV}^2\,, \\ \Sigma^{*\,2} - \Delta^2 \;&\approx\; \Xi^{*\,2} - \Sigma^{*\,2} \;\approx\; \Omega^2 - \Xi^{*\,2} \;\approx\; \frac{1}{2}\left(\Omega^2 - \Sigma^{*\,2}\right) &&\approx\; 0.43\,\mathrm{GeV}^2\,. \end{aligned} \tag{25}$$

However, the $\bar{q}q$ pseudoscalar and vector $\Delta S = 1$ ESLs have about one half this scale (also empirically valid to 3%), viz.

$$m_K^2 - m_\pi^2 \;\approx\; m_{K^*}^2 - m_\rho^2 \;\approx\; m_\phi^2 - m_{K^*}^2 \;\approx\; \frac{1}{2}\left(m_\phi^2 - m_\rho^2\right) \;\approx\; 0.22\,\mathrm{GeV}^2\,, \tag{26}$$

as roughly do the $\bar{q}q$ scalars found in Sec. 2, i.e.,

$$m_{\kappa(800)}^2 - m_{\sigma_n(650)}^2 \;\approx\; m_{\sigma_s(970)}^2 - m_{\kappa(800)}^2 \;\approx\; 0.22 - 0.30\,\mathrm{GeV}^2\,. \tag{27}$$

This approximate factor of 2 between Eqs. (25) and Eqs. (26,27) is because there are two $\Delta S = 1$ qqq transitions, whereas there is only one $\Delta S = 1$ transition for $\bar{q}q$ configurations.

So if we take Eq. (27) as physically meaningful, we may write

$$2m_\kappa^2 \;\approx\; m_{\sigma(600)}^2 + m_{f_0(980)}^2 \;\approx\; m_{\sigma_n(650)}^2 + m_{\sigma_s(970)}^2 \;\approx\; 1.32 - 1.36\,\mathrm{GeV}^2\,, \tag{28}$$

yielding $m_\kappa \approx 819$ MeV close to experiment, which again suggests these scalars are $\bar{q}q$ states.

These IMF quadratic mass schemes, along with the NJL-LσM κ mass $m_{\kappa(800)} = 2\sqrt{m_s \hat{m}} = 809$ MeV, again suggest (as do the empirical scales of Eqs. (26) and (27) vs. Eqs. (25)) that *all* ground-state meson nonets are $\bar{q}q$, whereas the baryon octet and decuplet are qqq states.

REFERENCES

1. K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002). See page 754 for non-$\bar{q}q$ states.
2. For quadratic-mass pseudoscalars ($\mathscr{P}$), the Gell-Mann–Okubo formula $m_\pi^2 + 3m_{\eta_8}^2 = 4m_K^2$ implies $m_{\eta_8} = 567$ MeV, 3% greater than the observed [1] $m_\eta = 547$ MeV. Then the η-η'-mixing sum rule $m_\eta^2 + m_{\eta'}^2 = m_{\eta_1}^2 + m_{\eta_8}^2$ requires $m_{\eta_1} = 947$ MeV, 3% greater than the $U(1)$-anomaly mass 917 MeV. For the $U(1)$ problem, see e.g. A. N. Patrascioiu and M. D. Scadron, Phys. Rev. D **22**, 2054 (1980). R. Delbourgo and M. D. Scadron, Phys. Rev. D **28**, 2345 (1983). S. R. Choudhury and M. D. Scadron, Mod. Phys. Lett. A **1**, 535 (1986). D. Kekez, D. Klabucar, and M. D. Scadron, J. Phys. G **26**, 1335 (2000) [arXiv:hep-ph/0003234].
3. E. M. Aitala *et al.* [E791 Collaboration], Phys. Rev. Lett. **89**, 121801 (2002) [hep-ex/0204018].
4. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
5. M. D. Scadron, F. Kleefeld, G. Rupp, and E. van Beveren, Nucl. Phys. A, in press (2003) [hep-ph/0211275].
6. E. van Beveren, F. Kleefeld, G. Rupp and M. D. Scadron, Mod. Phys. Lett. A **17**, 1673 (2002) [arXiv:hep-ph/0204139].
7. M. Gell-Mann and M. Lévy, Nuovo Cim. **16**, 705 (1960); V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, in *Currents in Hadron Physics*, North-Holland Publ., Amsterdam, Chap. 5 (1973).

8. R. Delbourgo and M. D. Scadron, Mod. Phys. Lett. A **10**, 251 (1995) [hep-ph/9910242].
9. A. Salam, Nuovo Cim. **25**, 224 (1962); S. Weinberg, Phys. Rev. **130**, 776 (1963); M. D. Scadron, Phys. Rev. D **57**, 5307 (1998) [hep-ph/9712425].
10. B. H. McKellar, M. D. Scadron, and R. C. Warner, Int. J. Mod. Phys. A **3**, 203 (1988).
11. M. D. Scadron, Phys. Rev. D **26**, 239 (1982); Mod. Phys. Lett. A **7**, 669 (1992).
12. E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J. E. Ribeiro, Z. Phys. C **30**, 615 (1986).
13. S. Fubini and G. Furlan, Physics **4**, 229 (1965).