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REMARKS ON THE f_0(400–1200) SCALAR MESON AS THE DYNAMICALLY GENERATED CHIRAL PARTNER OF THE PION

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The quark-level linear σ model (LσM) is revisited, in particular concerning the identification of the f_0(400–1200) (or σ(600)) scalar meson as the chiral partner of the pion. We demonstrate the predictive power of the LσM through the $\pi\pi$ and πN s-wave scattering lengths, as well as several electromagnetic, weak, and strong decays of pseudoscalar and vector mesons. The ease with which the data for these observables are reproduced in the LσM lends credit to the necessity to include the σ as a fundamental $q\bar{q}$ degree of freedom, to be contrasted with approaches like chiral perturbation theory or the confining NJL model of Shakin and Wang.

Keywords: f_0(400–1200) and pion as chiral partners; dynamical generation; quark-level linear sigma model.

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1. Introduction

The question as to whether the pion has a scalar $q\bar{q}$ partner remains highly topical, now that the f_0(400–1200) (or σ) meson has become a firmly established resonance.[1] For the latter reason, the bone of contention has shifted from the cavilling at the "existence" of the σ towards a somewhat more sensible discussion whether the σ

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is a "fundamental" or a "dynamically generated" particle. Now while there is little dispute about what "fundamental" (or "intrinsic") means in a mesonic context, namely a totally colorless state composed of normally one but possibly more $q\bar{q}$ pairs, the term "dynamically generated" (or "dynamical" only) has been used by several authors to express rather different physical mechanisms.

For instance, in a Comment[2] on a paper by Törnqvist and Roos (TR),[3] Isgur and Speth (IS) argued that the σ meson, at least in their approach, is a broad "dynamical pole" due to t-channel forces only, arising from degrees of freedom already present in the meson–meson continuum, in contrast to an "intrinsic pole" resulting from a new $q\bar{q}$ degree of freedom in the dynamics. Moreover, IS criticized and drew into question the conclusions of TR because of the omission of t-channel forces in their work. However, in another Comment[4] on TR's paper, Harada, Sannino and Schechter demonstrated in a concrete model calculation that this omission appears to be not very crucial and only mildly affects the σ-meson mass and width. Also in the unitarized meson model of two of us,[5] the σ resonance is a consequence of the inclusion of p-wave $q\bar{q}$ states, but strongly coupled to the meson–meson continuum via the $^3P_0$ mechanism. This gives rise to a doubling of the number of poles originally present in the ground-state confinement spectra, the lower poles corresponding to the light scalar mesons like the σ. In this formalism, it makes little sense to talk about "intrinsic" versus "dynamical" poles, since the whole unitarization scheme is highly dynamical, producing large effects that strongly influence all poles. A similar conclusion has been reached very recently by Boglione and Pennington.[6]

In chiral-symmetric approaches like the quark-level Linear σ Model (LσM)[7,8] and the Nambu–Jona-Lasinio (NJL) model,[9] the σ meson naturally appears as the chiral scalar $q\bar{q}$ partner of the pion. Moreover, in the LσM the σ, which is introduced as an elementary degree of freedom in the Lagrangian, is also self-consistently generated in loop order through a quark loop and tadpole.[8] So the σ meson is *both* "fundamental" *and* "dynamically generated". On the other hand, in the *confining* NJL model of Shakin and Wang (SW),[10,11] no light scalar $q\bar{q}$ state shows up, in contrast to the traditional NJL approach. However, SW do predict a light scalar resonance, which could be interpreted as the f_0(400–1200), merely through t- and u-channel ρ exchange in $\pi\pi$ scattering,[11] in much the same way as IS[2] (see above). Such states SW call "dynamically generated" resonances, as opposed to "pre-existing" ones. These model results have led them to conclude that[10] "*the σ obtained from the study of $\pi\pi$ scattering is not the chiral partner of the pion*" and "*the nonlinear sigma model is the model of choice*". In Ref. 11, SW also arrive at several other conclusions on the nature of different scalar mesons, which we have shown[12] to be not supported by experiment (see also Ref. 13). In Ref. 12, we also argued against the strict distinction between "intrinsic" and "dynamically generated" scalar–meson states made by SW.

In this paper, we re-address the issue of the pion's chiral $q\bar{q}$ partner, and reach the following conclusions:

(a) indeed an $f_0(630)$ is dynamically generated from the chiral field theory constituted by the quark-level LσM,[8] which is based on the original chiral Gell-Mann–Lévy nucleon LσM,[7] but also predicts the famous NJL[9] result $m_\sigma = 2m_q$;

(b) the above SW conclusion on the nature of the σ meson is incorrect. Instead, this $\sigma(630)$ is indeed the scalar $n\bar{n}$ chiral partner of the pion.

Rather than just repeating the analysis of Ref. 8, in Sec. 2 we demonstrate the chiral structure of these LσM states for strong, electromagnetic (e.m.), and weak interactions. In Sec. 3, we verify our use of chiral LσM couplings by showing that the corresponding P$\gamma\gamma$, VPγ plus PVγ, VPP e.m., and strong LσM quark loop-order graphs are always compatible with observed[1] SU(2) and SU(3) sum-rule data. We draw our conclusions in Sec. 4 and, in passing, note that both the SU(3)-symmetry and infinite-momentum-frame approaches of Ref. 14, and also the dynamical unitarized nonet scheme of Ref. 5, arrive at *different* $q\bar{q}$ patterns for the isoscalar scalar mesons than SW in Refs. 10 and 11.

2. Why the $f_0(630)$ Scalar σ Meson is the Chiral Partner of the π

2.1. *Brief summary of the LσM field theory*

The chiral-symmetric SU(2) LσM was first formulated in 1960,[7] while the SU(3) version dates from 1967, 1969 and 1971, respectively.[15] The LσM pseudoscalar and scalar nonet U(3) states [$\pi(140)$, $K(492)$, $\eta(549)$, $\eta'(958)$, and $\sigma(650)$, $\kappa(800–900)$, $f_0(980)$, $a_0(980)$] were later dynamically generated.[14,8] A LσM is manifestly renormalizable and much easier to handle than the nonlinear NJL scheme,[9] yet chiral symmetry in fact blends together these two pictures,[16] as the dynamically generated theory[8] shows. Specifically, the SU(2) LσM interaction Lagrangian — due to dynamical symmetry breaking[8] or spontaneous symmetry breaking[14] — reads, after the shift of the σ field,

$$\mathcal{L}^{\rm int}_{{\rm L}\sigma{\rm M}} = g\bar{\psi}(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi + g'\sigma(\sigma^2+\pi^2) - \frac{\lambda}{4}(\sigma^2+\pi^2)^2\,. \tag{1}$$

Here, the fermion fields refer to quarks[8] and not to nucleons, with constituent quark mass $m_q = (m_u + m_d)/2$ generated via the chiral Goldberger–Treiman relation (GTR) $f_\pi g = m_q$, with $f_\pi \approx 93$ MeV (and 90 MeV in the chiral limit (CL)[a]), resulting in a value near $m_q \approx m_N/3 \approx 315$ MeV. In fact, it is dynamically

[a]The once-subtracted dispersion-relation result

$$1 - \frac{f_\pi^{\rm CL}}{f_\pi} \approx \frac{m_\pi^2}{8(\pi f_\pi)^2} \approx 0.03$$

and $f_\pi \approx 93$ MeV imply $f_\pi^{\rm CL} \approx 90$ MeV, which can be found e.g. in: S. A. Coon and M. D. Scadron, *Phys. Rev.* **C23**, 1150 (1981).

generated in the CL as $m_q \approx 325$ MeV,[8] and the Gell-Mann–Lévy chiral relations at tree level are[7]

$$g = \frac{m_q}{f_\pi}\,, \qquad g = \frac{m_\sigma^2}{2f_\pi} = \lambda f_\pi\,. \tag{2}$$

Moreover, at one-loop level Eqs. (2) are recovered, together with two new equations[8] in the CL:

$$m_\sigma = 2m_q\,, \qquad g_{\pi qq} = g = \frac{2\pi}{\sqrt{N_c}}\,, \tag{3}$$

for $N_c = 3$, also dynamically generated. Then, $g = 2\pi/\sqrt{3} = 3.6276$, and

$$m_q = 2\pi\frac{f_\pi^{\rm CL}}{\sqrt{3}} \approx 325 \text{ MeV}\,, \qquad m_\sigma = 2m_q \approx 650 \text{ MeV} \tag{4}$$

are dynamically generated, from the chiral GTR.[8] Finally, all three LσM couplings in Eq. (1) are dynamically generated as

$$g = \frac{2\pi}{\sqrt{3}} \approx 3.6\,, \qquad g' = 2gm_q \approx 2.3 \text{ GeV}\,, \qquad \lambda = \frac{8\pi^2}{3} = 26.3\,. \tag{5}$$

Furthermore, this LσM then also recovers the vector-meson-dominance (VMD) prediction $g_{\rho\pi\pi} = g_\rho$ from quark loops alone. When the π-σ-π LσM meson loop is added, this VMD prediction is extended to[17]

$$\frac{g_{\rho\pi\pi}}{g_\rho} = \frac{6}{5} = 1.2\,. \tag{6}$$

Underlying Eqs. (3)–(6) is the CL log-divergent gap equation (LDGE)

$$1 = -i4N_c g^2 \int \frac{d^4p}{(2\pi)^4}(p^2 - m_q^2)^{-2}\,, \tag{7}$$

corresponding to the $V\pi\pi$ quark-loop form factors, automatically normalized to[8,18] $F_\pi(q^2 = 0) = 1$. Further invoking the LDGE (2.7) in turn requires[8,19]

$$g_{\rho\pi\pi} = \sqrt{3}g_{\pi qq} = 2\pi\,, \tag{8}$$

close to the value 6.04 needed to obtain the observed ρ width 150.2 MeV.

2.2. *Chiral cancellations for strong-interaction s-wave $\pi\pi$ and πN scattering lengths*

Consider the low-energy $\pi\pi$ and πN LσM graphs of Figs. 1 and 2. Away from the CL, the $\pi\pi$ contact graph with coupling λ (Fig. 1(a)) is related to the cubic meson coupling (Fig. 1(b)) as

$$g_{\sigma\pi\pi}(= g') = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi} = \lambda f_\pi\,. \tag{9}$$

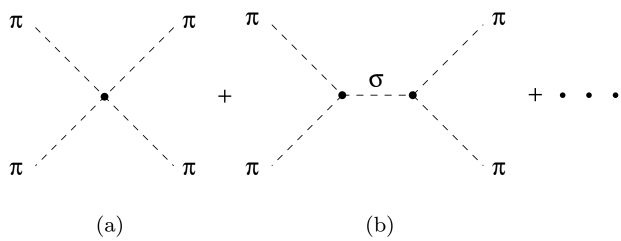

Fig. 1. Low-energy $\pi\pi$ LσM graphs.

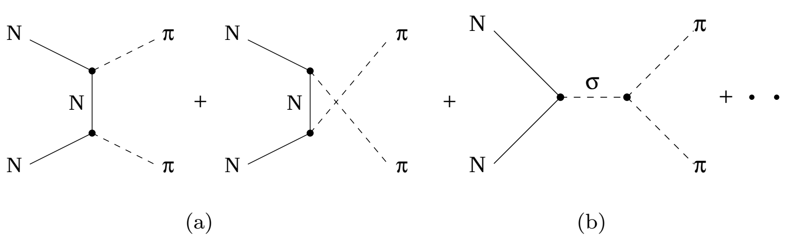

Fig. 2. Low-energy πN LσM graphs.

Then, at the soft-pion point $s = m_\pi^2$, the net $\pi\pi$ amplitude (Figs. 1(a) and (b)) "miraculously" vanishes[7]:

$$\mathcal{M}_{\pi\pi} = \mathcal{M}_{\pi\pi}^{\text{contact}} + \mathcal{M}_{\pi\pi}^{\sigma\text{-pole}} \to \lambda + 2g_{\sigma\pi\pi}^2 (m_\pi^2 - m_\sigma^2)^{-1} = 0\,. \tag{10}$$

In other words, the contact term λ "chirally eats" the σ pole at $s = m_\pi^2$, due to Eq. (9). Crossing symmetry then extends Eq. (10) to the Weinberg PCAC form,[20] but generalized to the LσM[21]:

$$\begin{aligned} \mathcal{M}_{\pi\pi}^{abcd} &= A\delta^{ab}\delta^{cd} + B\delta^{ac}\delta^{bd} + C\delta^{ad}\delta^{bc}\,, \\ A^{\text{L}\sigma\text{M}} &= -2\lambda\left[1 - \frac{2\lambda f_\pi^2}{m_\sigma^2 - s}\right] = \frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\frac{s - m_\pi^2}{f_\pi^2}\,. \end{aligned} \tag{11}$$

Thus, for $m_\sigma = 650$ MeV (as dynamically generated in Ref. 8 via the GTR), the $I = 0$ s-channel amplitude $3A+B+C$ predicts a 23% enhancement of the Weinberg s-wave scattering length at $s = 4m_\pi^2$, $t = u = 0$, $\varepsilon = m_\pi^2/m_\sigma^2 = 0.045$:

$$a_{\pi\pi}^{(0)}|_{\text{L}\sigma\text{M}} = \frac{7+\varepsilon}{1-4\varepsilon}\frac{m_\pi}{32\pi f_\pi^2} \approx 1.23\frac{7m_\pi}{32\pi f_\pi^2} \approx 0.20\, m_\pi^{-1}\,. \tag{12}$$

If instead we use $m_\sigma = 550$ MeV, a value which is closer to what is found in unitarized meson models,[5,3] we get $\varepsilon = 0.063$, so that Eq. (12) yields an increased scattering length $a_{\pi\pi}^{(0)}|_{\text{L}\sigma\text{M}} \approx 0.22\, m_\pi^{-1}$. The latter result is also obtained in a two-loop chiral-perturbation-theory (ChPT) calculation involving about 100 arbitrary LECs! So we prefer working with the simple parameter-free LσM form (2.12), since the Weinberg PCAC scattering length[20] is based on the PCAC equation itself, first derived via the LσM Lagrangian,[7] our Eq. (1).

Proceeding to the s-wave πN scattering length, the πN background amplitude with pseudoscalar (PS) coupling and "Adler consistency condition" (ACC) is,[22] for $q \to 0$,

$$\mathcal{M}^{ij}_{\pi N}(\mathrm{PS}) = \frac{g^2_{\pi NN}}{m_N}\delta^{ij}\,. \tag{13}$$

Then the isospin-zero scattering length corresponding to the "large" PS πN pole term, reads

$$a^{(+)}_{\pi N}(\mathrm{PS}) = -\frac{g^2_{\pi NN}}{4\pi}\frac{1}{m_N + m_\pi} \approx -1.8\, m_\pi^{-1}\,, \tag{14}$$

is reduced to near zero by adding to it the term of Eq. (13):

$$a^{(+)}_{\pi N}(\mathrm{Adler}) = -\frac{g^2_{\pi NN}}{4\pi}\frac{m^2_\pi/4m^2_N}{m_N + m_\pi} \approx -0.01\, m_\pi^{-1}\,, \tag{15}$$

due to the ACC soft-pion theorem. Stated in LσM language, when the σ pole in Fig. 2(b) is added to Fig. 2(a) (Eq. (14)), the net πN scattering length (due to the LσM coupling (2.9)) combined with the GTR again leads to the small scattering length (2.15).[b,23]

These "miraculous"[7] chiral cancellations, Eqs. (10) and (15), both due to the LσM coupling (2.9), appear to follow the experimental data, suggesting $a^{(+)}_{\pi N} \approx -0.005\, m_\pi^{-1}$ back in 1979,[24] and now finding[25] $a^{(+)}_{\pi N} = (-0.0001\,^{+0.0009}_{-0.0021})\, m_\pi^{-1}$ and $a^{(+)}_{\pi N} = (-0.22 \pm 0.43)\, m_\pi^{-1}$, respectively. However, ChPT advocates prefer to work with a (seemingly non-renormalizable and obviously nonlocal) pseudovector theory, derived from a nonlinear σ model, from which the σ meson has been eliminated as a fundamental degree of freedom. In our opinion, this is one of the reasons why in ChPT the above results require such a tremendous effort, while they are almost trivially obtained in the quark-level LσM. At this point, we also cannot ignore the mounting experimental evidence for the existence of the σ.[1]

[b] The explicit analytic expression for the isoscalar πN scattering length obtained by the sum of the pole term and the sigma exchange can be found, e.g., in Chap. 10.4 of the book, J. I. Kapusta, *Finite-Temperature Field Theory* (Cambridge Univ. Press, 1989). In our notation it is given by:

$$a^{(+)}_{\pi N} = \frac{1}{4\pi}\frac{1}{(1 + (m_\pi/m_N))}\left(\underbrace{\frac{2g_A(0)g' g_{\sigma NN}}{m^2_\sigma}}_{\text{"}\sigma\text{-exchange"}} - \underbrace{\frac{g^2_{\pi NN}}{m_N[1-(m_\pi/2m_N)^2]}}_{\text{"pole-term"}}\right)\,. \tag{16}$$

The resulting isoscalar scattering length is consistent with our estimate obtained from the ACC given in Eq. (15). Furthermore, we should note that various previous works treating πN scattering on the basis of the sum of a pole- and a sigma-exchange term (like e.g. the model of Hamilton[23]) associate the pole term with the excitation of intermediate antinucleons rather than nucleons.

2.3. *Pion charge radius and the chiral pion*

Now we comment on the chiral structure of the pion charge radius[26]

$$r_\pi^2 = 6\frac{dF_\pi(q^2)}{d(q^2)}\bigg|_{q^2=0} = \frac{3}{4\pi^2 f_\pi^2} = (0.60\ \text{fm})^2 \tag{17}$$

for the chiral-limiting value $f_\pi = 90$ MeV, which result is close to the measured[27] 0.63 ± 0.01 fm. Invoking the LσM relation[8] $f_\pi^{\text{CL}} = \frac{\sqrt{3}}{2\pi} m_q$ from Eq. (4) above, then Eq. (17) requires, from the quark-loop pion form factor at $q^2 = 0$,[17]

$$r_\pi = \frac{1}{m_q} = 0.61\ \text{fm}\,. \tag{18}$$

This tightly bound (fused) pion charge radius, as observed experimentally, certainly suggests the chiral pion wave function is $q\bar{q}$. Note that ChPT requires r_π to be proportional to the parameter "L_9".[c] We prefer the parameter-free forms, Eqs. (17) and (18) above.

2.4. *Chiral couplings for* $\pi^0 \to 2\gamma$ *and* $\sigma \to 2\gamma$ *e.m. decays*

One knows that PVV LσM coupling[28] or AVV coupling[29] gives the gauge-invariant chiral quark-loop $\pi^0 \to 2\gamma$ amplitude

$$|F_{\pi^0\to 2\gamma}| = \frac{\alpha}{\pi f_\pi} \approx 0.025\ \text{GeV}^{-1}\,, \tag{19}$$

for $N_c = 3$. This is in perfect agreement with the data[1] $\Gamma_{\pi^0\to\gamma\gamma} = m_\pi^3 |F_{\pi^0\to 2\gamma}|^2/64\pi$ or $|F_{\pi^0\to 2\gamma}| = 0.025 \pm 0.001\ \text{GeV}^{-1}$, for $N_c = 3$. Likewise, the chiral partner to the π, the $\sigma(630)$, predicts the gauge-invariant quark-loop-plus-π^+-loop amplitude[30]

$$|F_{\sigma\to\gamma\gamma}| = \frac{5}{3}\frac{\alpha}{\pi f_\pi} + 0.5\frac{\alpha}{\pi f_\pi} \approx 2.2\frac{\alpha}{\pi f_\pi} \approx 0.055\ \text{GeV}^{-1}\,, \tag{20}$$

corresponding to the decay rate (for $m_\sigma = 630$ MeV)

$$\Gamma_{\sigma\to\gamma\gamma} = \frac{m_\sigma^3 |F_{\sigma\to\gamma\gamma}|^2}{64\pi} \approx 3.76\ \text{keV}\,. \tag{21}$$

This prediction is reasonably compatible with the extracted $\sigma \to 2\gamma$ rates[31,32] (3.8 ± 1.5) keV and (5.4 ± 2.8) keV, respectively, provided that these rates indeed refer to the σ, as advocated by the authors,[31] and not to the $f_0(1370)$.

[c] J. Gasser and H. Leutwyler, *Ann. Phys.* **158**, 142 (1984); *Nucl. Phys.* **B250**, 517 (1985). In the former paper, Appendix II attempts to rule out a σ resonance below 1 GeV in a LσM context. However, this "proof" is *not* based on the standard LσM of our Refs. 7 and 8 as summarized in Sec. 2 above. A recent strong-interaction argument favoring a σ-meson theory over ChPT (besides Refs. 17, 26 and 27) was given by J. Schechter, hep-ph/0112205.

2.5. *Chiral transitions for weak $K_S \to 2\pi$ decays*

The s-channel σ-pole graph of Fig. 3 dominates parity-violating (PV) $K_S \to 2\pi$ decays, with PV weak amplitude magnitude

$$|\langle 2\pi|H_w^{\rm PV}|K_S\rangle| = |2\langle 2\pi|\sigma\rangle| \frac{|\langle\sigma|H_w^{\rm PV}|K_S\rangle|}{m_{K_S}^2 - m_\sigma^2 + im_\sigma\Gamma_\sigma} \approx \frac{1}{f_\pi}|\langle\sigma|H_w^{\rm PV}|K_S\rangle|\,, \tag{22}$$

since $\langle 2\pi|\sigma\rangle_{\rm L\sigma M} = m_\sigma^2/2f_\pi$, $m_K \simeq m_\sigma$, and $\Gamma_\sigma \simeq m_\sigma$ for the broad σ meson.[1] However, pion PCAC consistency requires[33]

$$|\langle 2\pi|H_w^{\rm PV}|K_S\rangle| \to \frac{1}{f_\pi}|\langle\pi|[Q_5^\pi, H_w^{\rm PV}]|K_S\rangle| = \frac{1}{f_\pi}|\langle\pi|H_w^{\rm PC}|K_L\rangle|\,, \tag{23}$$

for H_w built up from $V - A$ chiral currents (PC = parity conserving). Equating (22) to (23) gives a definition of chiral π and σ partners[30]:

$$|\langle\sigma|H_w^{\rm PV}|K_S\rangle| = |\langle\pi^0|H_w^{\rm PC}|K_L\rangle|\,. \tag{24}$$

The charge algebra $[Q + Q_5, H_w] = 0$, PCAC, and Eq. (24) clearly suggest that the $\pi(140)$ and $\sigma(630)$ are chiral partners.

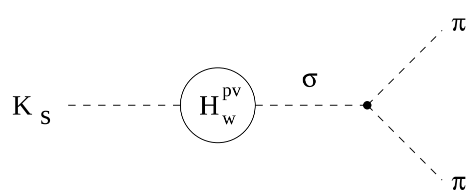

Fig. 3. Parity-violating decay $K_s \to \pi\pi$ via a sigma pole.

3. Quark-Loop LσM Strong and e.m. Decays

Rather than proceeding on with more detailed weak-interaction predictions, from Sec. 2 we test the LσM quark-loop predictions directly against the data for strong and e.m. decays. First consider the *udu* plus *dud* quark loops for $\rho^0 \to \pi^+\pi^-$ decay with the LDGE (2.7), leading to $g_{\rho\pi\pi} = 2\pi$, Eq. (8). The latter LσM VMD coupling predicts the rate

$$\Gamma_{\rho^0\to 2\pi} = \frac{g_{\rho\pi\pi}^2}{m_\rho^2}\frac{|\mathbf{p}|^3}{6\pi} = 162.6 \text{ MeV} \qquad \text{for } |\mathbf{p}| = 358 \text{ MeV}\,, \tag{25}$$

close to data at[1] (150.2±0.8) MeV. For the small $\rho^0 \to e^+e^-$ and $\omega \to e^+e^-$ decays, we use single-photon exchange to extract the g_ρ and g_ω couplings from data[1]:

$$\Gamma_{\rho^0\to e^+e^-} = \frac{\alpha^2}{3}m_\rho\frac{4\pi}{g_\rho^2} = 6.77 \pm 0.32 \text{ keV}\,, \tag{26}$$

$$\Gamma_{\omega\to e^+e^-} = \frac{\alpha^2}{3}m_\omega\frac{4\pi}{g_\omega^2} = 0.60 \pm 0.02 \text{ keV}\,, \tag{27}$$

leading to

$$g_\rho \approx 5.03\,, \qquad g_\omega \approx 17.05\,. \tag{28}$$

The latter couplings are near the U(3) value $g_\omega = 3g_\rho$, assuming the ω is purely non-strange. But one knows[1] that there is a slight ω–ϕ mixing angle $\phi_V \approx 3.7°$, from the small $\phi \to \pi\gamma$ decay. Note that the LσM coupling $g_{\rho\pi\pi}$ is relatively near $g_\rho \approx 5.03$ found in Eq. (28). However, when one adds the π-σ-π meson loop to the quark loop, one knows from the LσM Eq. (6) that actually $g_{\rho\pi\pi}/g_\rho = 6/5$, whereas Eq. (28) predicts the nearby ratio

$$\frac{g_{\rho\pi\pi}}{g_\rho} \approx \frac{2\pi}{5.03} \approx 1.25\,. \tag{29}$$

Next consider the e.m. decays $\rho \to \pi\gamma$ and $\omega \to \pi\gamma$. Our only use of SU(3) symmetry is $\lambda_\gamma = \lambda_3 + \lambda_8/\sqrt{3}$. This predicts the quark-loop decays, using the g_ρ and g_ω couplings from data in Eq. (28),

$$\Gamma_{\rho\to\pi\gamma} = \frac{|\mathbf{p}|^3}{12\pi}|\mathcal{M}_{\rho\pi\gamma}|^2 = 59 \text{ keV} \qquad \text{for } |\mathbf{p}| = 372 \text{ MeV}\,, \tag{30}$$

with $|\mathcal{M}_{\rho\pi\gamma}| = eg_\rho/8\pi^2 f_\pi = 0.207 \text{ GeV}^{-1}$, which comes out close to the data[1] $\Gamma_{\rho^\pm\to\pi^\pm\gamma} = (68 \pm 7)$ keV. Likewise, the LσM predicts

$$\Gamma_{\omega\to\pi\gamma} = \frac{|\mathbf{p}|^3}{12\pi}|\mathcal{M}_{\omega\pi\gamma}|^2 = 711 \text{ keV} \qquad \text{for } |\mathbf{p}| = 379 \text{ MeV}\,, \tag{31}$$

with $|\mathcal{M}_{\omega\pi\gamma}| = eg_\omega \cos\phi_V/8\pi^2 f_\pi = 0.7017 \text{ GeV}^{-1}$, very close to the data[1] $\Gamma_{\omega\to\pi\gamma} = (717 \pm 43)$ keV.

Finally, the e.m. decay $\pi^0 \to 2\gamma$ is predicted via u and d quark loops, together with the gauge-invariant amplitude (2.18) and $N_c = 3$, to be

$$\Gamma_{\pi^0\to 2\gamma} = \frac{|\mathbf{p}|^3}{8\pi}|\mathcal{M}_{\pi^0 2\gamma}|^2 = 7.64 \text{ eV} \qquad \text{for } |\mathbf{p}| = 67.49 \text{ MeV}\,, \tag{32}$$

again close to the data[1] $\Gamma_{\pi^0\to 2\gamma} = (7.74 \pm 0.55)$ eV.

When considering η and η' initial states, we circumvent explicit η–η' mixing by only computing the sum of their squared matrix elements, thereby using $\cos^2\phi_{PS} + \sin^2\phi_{PS} = 1$. Then, Table I of Ref. 28 shows the PVV quark-loop matrix elements for $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, $\eta' \to 2\gamma$ are, respectively, A, $A(5\cos\phi_{PS} - \sqrt{2}r_s \sin\phi_{PS})/3$, $A(5\sin\phi_{PS} + \sqrt{2}r_s\cos\phi_{PS})/3$, for $A = \alpha/\pi f_\pi \approx 0.025 \text{ GeV}^{-1}$, and where $r_s = \hat{m}/m_s \approx 1/1.44$ is the constituent-quark-mass ratio, with $m_s/\hat{m} = 2f_K/f_\pi - 1$ and $f_K/f_\pi = 1.22$. Therefore, the matrix-element squares satisfy $|\mathcal{M}_{\eta 2\gamma}|^2 + |\mathcal{M}_{\eta' 2\gamma}|^2 = A^2(25 + 2/1.44^2)/9$, corresponding to the L$\sigma$M decay-rate SU(3) sum rule, implied from Ref. 28,

$$\frac{\Gamma_{\eta 2\gamma}}{m_\eta^3} + \frac{\Gamma_{\eta' 2\gamma}}{m_{\eta'}^3} = 2.885\frac{\Gamma_{\pi^0 2\gamma}}{m_{\pi^0}^3}\,. \tag{33}$$

Given the measured central-value rates and masses[1] $\Gamma_{\eta 2\gamma} = 464$ eV, $m_\eta = 0.5473$ GeV, $\Gamma_{\eta' 2\gamma} = 4282$ eV, $m_{\eta'} = 0.9578$ GeV, $\Gamma_{\pi^0 2\gamma} = 7.74$ eV, $m_{\pi^0} =$

0.1349766 GeV, the L.H.S. of Eq. (33) sums to 7704×10^{-9} GeV^{-2}, while the R.H.S. is 9081×10^{-9} GeV^{-2}. A one-standard-deviation reduction of the R.H.S. gives 8435×10^{-9} GeV^{-2}, only 9% greater than the L.H.S. of Eq. (33).

Likewise, we can construct an SU(3) sum rule again by invoking $\sin^2\phi + \cos^2\phi = 1$ for any angle, and referring to Ref. 28 for $\eta' \to \rho\gamma$, $\rho \to \eta\gamma$, $\rho \to \pi\gamma$ decays. The squares of the LσM quark-loop matrix elements are $|\mathcal{M}_{\eta'\rho\gamma}|^2 + |\mathcal{M}_{\rho\eta\gamma}|^2 = 9B^2$ and $|\mathcal{M}_{\rho\pi\gamma}|^2 = B^2$, corresponding to the L$\sigma$M decay-rate SU(3) sum rule[28]

$$\frac{\Gamma_{\eta'\rho\gamma}}{|\mathbf{p}_1|^3} + 3\frac{\Gamma_{\rho\eta\gamma}}{|\mathbf{p}_2|^3} = 27\frac{\Gamma_{\rho\pi\gamma}}{|\mathbf{p}_3|^3}\,. \tag{34}$$

For the measured central-value rates and CM momenta[1] $\Gamma_{\eta'\rho\gamma} = 59.6$ keV, $|\mathbf{p}_1| = 169$ MeV; $\Gamma_{\rho\eta\gamma} = 36.05$ keV, $|\mathbf{p}_2| = 189$ MeV; $\Gamma_{\rho\pi\gamma} = 67.6$ keV, $|\mathbf{p}_3| = 372$ MeV, the L.H.S. of the sum rule Eq. (34) sums up to 1.2348×10^{-2} GeV^{-2} + 1.6019×10^{-2} GeV^{-2} = 2.8367×10^{-2} GeV^{-2}, while the R.H.S. is 3.5455×10^{-2} GeV^{-2}. Considering we have combined the PS η' decay rate and two vector ρ decay rates, we suggest that the LσM SU(3) sum rule (3.10) is reasonably well satisfied.

By analogy with Eqs. (33) and (34), another SU(3) sum rule implied in Ref. 28 reads

$$\frac{\Gamma_{\eta'\omega\gamma}}{|\mathbf{p}_a|^3} + 3\frac{\Gamma_{\omega\eta\gamma}}{|\mathbf{p}_b|^3} = 0.336\frac{\Gamma_{\omega\pi\gamma}}{|\mathbf{p}_c|^3}\,. \tag{35}$$

From Ref. 1, the measured central-value rates and CM momenta are $\Gamma_{\eta'\omega\gamma} = 6.12$ keV, $|\mathbf{p}_a| = 160$ MeV; $\Gamma_{\omega\eta\gamma} = 5.486$ keV, $|\mathbf{p}_b| = 199$ MeV; $\Gamma_{\omega\pi\gamma} = 717$ keV, $|\mathbf{p}_c| = 379$ MeV. Then the L.H.S. of Eq. (35) sums to 1.4941×10^{-3} GeV^{-2} + 2.0884×10^{-3} GeV^{-2} = 3.5825×10^{-3} GeV^{-2}, while the R.H.S. is nearby at 4.4253×10^{-3} GeV^{-2}. If one increases the $\omega \to \eta\gamma$ rate by one standard deviation, the L.H.S. of Eq. (35) becomes 3.940×10^{-3} GeV^{-2} — again only 9% below the R.H.S.

4. Conclusions

In the preceding we have shown, by straightforward computation, that the quark-level LσM of Refs. 14 and 8 easily reproduces the small $\pi\pi$ and πN s-wave scattering lengths, the pion charge radius, and a variety of e.m., weak, and strong decays of pseudoscalar and vector mesons. The crucial part for most of these processes is the inclusion of the f_0(400–1200), alias σ meson, as a fundamental $q\bar{q}$ degree of freedom. This occurs very naturally in the LσM, where the σ can then also be dynamically and self-consistently generated,[8] as well as in the unitarized quark/meson model of Ref. 5. Moreover, a finite-temperature (recall footnote b) chiral-phase-transition approach,[34] which independently "melts" the quark mass, the σ mass, and the quark condensate in QCD, suggests that the above LσM can be identified as the infrared limit of QCD.[35]

In contrast, nonlinear approaches where the σ does not show up as a $q\bar{q}$ state or has even been designedly eliminated as a fundamental degree of freedom, like the confining NJL-type model of SW[10,11] and ChPT, appear to have difficulties in reproducing several low-energy data, besides having strained relations with the now firmly established σ itself.[c,30] We therefore argue that the conclusion of SW[10] according to which the σ is *not* the chiral partner of the pion is not based on "*major chiral-symmetry violations*", but rather on the complications and possible approximations in their nonlinear NJL scheme. In this respect, we should point out the following apparent contradiction in SW's line of reasoning. In Ref. 10 they conclude that confinement is quite a small effect for the $\pi(138)$ and $K(495)$ mesons, which may even be best to neglect altogether. However, in Sec. 2.3 we showed that the observed pion charge radius suggests in fact a $q\bar{q}$ (fused) π meson composed of tightly bound quarks, corresponding to an almost massless Nambu–Goldstone pion. Moreover, the well-understood NJL model *without* confinement *does* predict a bound-state σ meson as the chiral $q\bar{q}$ partner of the pion. We believe to have demonstrated, in the framework of the quark-level LσM, that this is indeed the scenario favored by experiment.

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